Mathematical Appendix

This appendix is intended for use by mathematically trained persons. If you are not a mathematician, do not feel that you will be left behind because you do not understand it. If you have carefully read the preceding chapters you should already have a firm picture of all the important concepts and ideas without added mathematics. On the other hand, mathematical relationships are important to help pioneers on the road to further exploration, as well as to establish with certainty some of the ideas which are only stated in the text.

Most of the mathematical proofs below are separated subjects which stand alone. They have the same bold-face headings as the related material of the text.

The Particle Waves Assumption

The first assumption of this model is that the waves of the space resonances are solutions of a scalar wave equation whose waves propagate in space with velocity c. This is an assumption that *space* has this property of propagating particle waves. There is no proof that this is true, but it can be believed if the results are more in agreement with experiments than are other theories.

I. PARTICLE-WAVES ASSUMPTION:

Space can propagate scalar waves, not directly observable, according to

$$\nabla^2\Phi - \frac{1}{c^2}\left(\frac{\partial^2\Phi}{\partial t^2}\right) = 0$$

where Φ is a continuous scalar amplitude with values everywhere in space, and c is the propagation speed.

This equation is similar to many other oscillatory equations found in nature. The assumption provides only a propagation equation for the particle waves.

Two solutions of the equation, an inward-moving spherical "IN wave," and an outward-moving "OUT wave" are combined to form a standing wave whose properties are investigated in this paper. This combination is a "space-resonance."

An important result is that the scalar wave-equation allows the amplitude to be finite at the center. Mathematically, this is only possible for a scalar wave, not a vector wave .

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Spherical Wave Solutions

The two spherical wave solutions which form the space resonances will be obtained now for the case of a stationary resonance.

The wave equation when written in spherical coordinates, becomes

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\left(\frac{\partial \Phi}{\partial r}\right) - \frac{1}{c^2}\left(\frac{\partial^2}{\partial t}\right) = 0$$

where Φ is wave amplitude and **r** is radial distance.

This equation has two solutions for the amplitude Φ; one of them is a converging (IN) spherical wave and the other is a diverging (OUT) wave,

$$\Phi^{IN} = \frac{1}{r}\Phi_0 e^{(i\omega t + i\kappa r)} \quad , \quad \Phi^{OUT} = \frac{1}{r}\Phi_0 e^{(i\omega t - i\kappa r)}$$

They can be combined so that the amplitudes at r = 0 are opposite. This combination removes the infinity at r = 0. The combined wave is the difference of the amplitudes of the IN and OUT waves of a resonance, which is written

$$\Phi = \Phi^{IN} - \Phi^{OUT} = Ae^{(i\omega_i t + i\kappa_i r)} - Ae^{(i\omega_o t - i\kappa_o r)}, \qquad i = IN,\ o = OUT$$

where $\omega = 2\pi mc^2/h = \kappa c$ is the mass-frequency of the space resonance. The complex amplitudes A include the range factor of 1/r and are alike because of the symmetry of the IN and OUT waves.

The intensity of combined IN and OUT waves is the envelope of $\Phi^*\Phi$,

$$\Phi^*\Phi = \left(Ae^{-(i\omega_i t + i\kappa_o r)} - Ae^{-(i\omega_i t - i\kappa_o r)}\right) \times \left(Ae^{(i\omega_i t + i\kappa_o r)} - Ae^{(i\omega_i t - i\kappa_o r)}\right)$$

After multiplying and reducing, the intensity becomes,

$$\Phi^*\Phi = 2A^2 - 2A^2\cos\left[(\kappa_i + \kappa_o)r - (\omega_i - \omega_o)t\right]$$

The distinction between frequencies and wave numbers of the two waves can be used to investigate different properties, but if there is no motion of the resonance, then $\omega_i = \omega_o = \omega$ and $\kappa_i = \kappa_o = \kappa$, and the intensity reduces to

$$\Phi^*\Phi = 4A^2\left[\tfrac{1}{2} - \tfrac{1}{2}\cos(2\kappa r)\right] = \left[2A\sin(\kappa r)\right]$$

which is the envelope of the oscillating standing particle waves. The standing wave has nodes located at $r = n\,\pi/\kappa$. It has spherical symmetry in its own inertial frame, and may be envisioned like layers of spherical, concentric, oscillating nodes whose intensity decreases as $1/r^2$ away from its center.

The intensity at the center is obtained by taking the limit as $r \to 0$ in the sine function and in A. It is equal to the constant part of A, so the absurdity of

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the point charge of the electron leading to an infinite energy does not occur. The standing waves in this spherical geometry are mathematically analogous to standing waves in a long pipe, except that the pipe has undergone a transformation that opens one end to 4π radians (becomes a sphere) while stretching the radius to ∞, and at the same time shrinking the other end of the pipe to a point at the origin.

Two Resonances With Relative Motion

Relative motion between two resonances is very important since both QM and special relativity are physical laws which depend upon the relative velocity. Accordingly we now investigate the properties of the resonance waves which arrive from another resonance having relative velocity $\beta = v/c$ with respect to the first.

The frequency of the resonances have been chosen to be equal to the mass-frequency of a fundamental particle like an electron. By doing this, the relation between a space resonance and a particle can be quickly seen. This is an assumption, or can be regarded as incorporating experimental measurement of masses into the theory.

The appearance of the waves from a resonance, which arrive at another resonance, are changed if relative motion with velocity $\beta = v/c$ exists. Then the Doppler effect alters the received frequencies, velocities, and wave numbers. The IN waves are red shifted and the OUT waves are blue shifted according to the relativistic Doppler factors, D and 1/D, where

$$D = \gamma(1-\beta), \quad 1/D = \gamma(1+\beta), \quad \text{and} \quad \gamma = (1-\beta^2)^{-1/2}$$

The effect is perfectly symmetrical as it must be in relativity. Both resonances receive the same information from the other because the relative velocity is the same for both.

Note that we are only calculating waves which pass by and through each resonance. There is no reception or interaction by either resonance, since we have not yet introduced any means of energy exchange.

Using these factors, the received Doppler-shifted wave amplitude is then

$$\Phi = Ae^{i(ct+r)\kappa/D} - Ae^{i(ct-r)\kappa D}$$

Inserting the expressions for the Doppler factors,

$$\Phi = Ae^{i(ct+r)\kappa/\gamma(1+\beta)} - Ae^{i(ct-r)\kappa\gamma(1-\beta)}$$

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Multiplying exponents, rearranging, and factoring,

$$\Phi = Ae^{i\kappa\gamma(ct+\beta r)}\left(e^{i\kappa\gamma(\beta ct+r)} - e^{-i\kappa\gamma(\beta ct+r)}\right)$$

Combining the last two exponential terms, the Doppler-shifted wave of either resonance, received at the other resonance, becomes

$$\Phi = 2Ae^{i\kappa\gamma(ct+\beta r)}\sin\left[\kappa\gamma(\beta ct + r)\right]$$

This equation has the form of an exponential carrier wave modulated by a sinusoid. The surprising characteristics of the carrier wave are:

wavelength = $h/\gamma mv$ = deBroglie wavelength

frequency = $k\gamma c/2\pi = \gamma mc^2/h$ = mass-energy frequency

velocity = c/β = phase velocity.

The modulating sine function has:

wavelength = $h/\gamma mc$ = Compton wavelength

frequency = $\gamma mc^2\beta/h = \beta$ (mass frequency) = momentum frequency

velocity = $\beta c = v$ = relative velocity of the two resonances.

These wave properties are complete. They show that the two resonances contain the law of special relativity, and the law of QM implied by the deBroglie wavelength. All the parameters that can be measured for a moving particle, are contained in the above equation. Respectively, they are: The two quantum-mechanical parameters: a deBroglie wavelength and the Compton wavelength; and for relativity, the elements of the four-momentum vector: i.e. rest mass and three components of linear momentum. The latter are expressed in terms of frequency with the correct Lorentz factors.

Because the resonances contain the deBroglie wavelength, it can be used to obtain the Schroedinger Equation as originally constructed by Schroedinger.

Magnetic Equations from Coulomb's Law and Relativity

When Maxwell propounded his four equations in 1886, they were heralded as the fundamental expression of electromagnetic laws. Later, 1905 and later, Einstein's special relativity was discovered, but it was not significantly recognized for about a half-century that the two laws of induction

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involving magnetism were a consequence of relative motion between charges, or that relativity played a role.

Even today, few scientists are fully aware that magnetic forces are a perturbation of the Coulomb electric forces as a result of relative motion. Maxwell's Equations, despite their fundamental value in the study of electricity, are not fundamental laws of Nature. Only the Coulomb force law and the special relativity are fundamental. Maxwell's Equations are obtained from these two.

In order to make it clear that Maxwell's magnetic equations do not belong on the list of fundamental laws, I show below how they are obtained from Coulomb's law and relativity.

Transformations of Special Relativity

We will use transformations between a reference frame 1 and a reference frame 2 which have a relative velocity v in the x direction. Subscripts 1 and 2 on the symbols indicate that they represent quantities measured by observers located in frame 1 and frame 2.

The relativistic expansion factor γ is

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Charge and Current Density. Charge density ρ_2 which is stationary in frame 2 is seen as a current density J_{1x} moving in the x direction in frame 1, and vice-versa. The transformation is

$$\rho_1 = \gamma\left[\rho_2 + \left(\frac{v}{c^2}\right) J_{2x}\right] \qquad \text{(A-1)}$$

Partial Derivatives. The operations of taking derivatives in one frame transform to the other frame as

$$\frac{\partial}{\partial x_1} = \gamma\left[\frac{\partial}{\partial x_2} - \left(\frac{v}{c^2}\right)\frac{\partial}{\partial t}\right]$$

$$\frac{\partial}{\partial y_1} = \frac{\partial}{\partial y_2}\ , \quad \frac{\partial}{\partial z_1} = \frac{\partial}{\partial z_2} \qquad \text{(A-2)}$$

Components of Electric Field E.

$$E_{x2} = E_{x1}$$
$$E_{y2} = \gamma\left(E_{y1} - vB_{z1}\right) \qquad \text{(A-3)}$$
$$E_{z2} = \gamma\left(E_{z1} - vB_{y1}\right)$$

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Force Transformations ($v_2 = 0$)

$$F_{x1} = F_{x2} \qquad F_{y1} = \frac{F_{y2}}{\gamma} \qquad F_{z1} = \frac{F_{z2}}{\gamma} \tag{A-4}$$

Lorentz Transformations:

$$x_2 = \gamma(x_1 - vt_1)$$
$$y_2 = y_1 \qquad \text{and} \qquad z_2 = z_1 \tag{A-5}$$

How to Derive Maxwell's Curl B Equation

We want to show that the equation of electric induction

$$\nabla \times B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\left(\frac{\partial E}{\partial t}\right)$$

can be obtained from equations (A-1, A-2, and A-3) and Coulomb's law.

Choose that the density of charge ρ_2, is stationary in frame 2, moves with velocity v with respect to frame 1. Therefore in frame 2, the magnetic field B = 0.

From eqn. (A-1), we see that $\rho_2 = \rho_1/\gamma$.

Using Gauss's law (vector form of Coulomb's Law), $\nabla \cdot E_2 = r_2/e_o = r_1/e_o\gamma$, we can write in cartesian coordinates,

$$\frac{\partial E_{x2}}{\partial x_2} + \frac{\partial E_{y2}}{\partial y_2} + \frac{\partial E_{z2}}{\partial z_2} = \frac{\rho_1}{\gamma \varepsilon_0}$$

Then from the transformation equations (A-2) for derivatives and from (A-3) for E components, we can find the field in frame 1,

$$\nabla E_1 + \frac{v}{c^2}\left(\frac{\partial E_{x1}}{\partial t}\right) - v\left(\frac{\partial B_{z1}}{\partial y} - \frac{\partial B_{y1}}{\partial z}\right) = \rho_1 \frac{(1-\beta^2)}{\varepsilon_0}$$

We can replace the first term using, $\nabla . E_1 = \rho_1/\varepsilon_o$, which cancels the next to last term, and then divide by v to get,

$$\frac{1}{c^2}\left(\frac{\partial E_{x1}}{\partial t}\right) - \left(\frac{\partial B_{z1}}{\partial y} - \frac{\partial B_{y1}}{\partial z}\right) = \rho 1 \frac{v}{c^2 \varepsilon_0}$$

The term in brackets is the x component of curl $B_1 = \nabla \times B_1$. Corresponding terms for the y and z components can be obtained by rotating indices. Adding

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all three components, replacing $\rho_1 v$ by its equivalent J_1, dropping the subscripts 1, and rearranging terms, we obtain

$$\nabla \times B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\left(\frac{\partial E}{\partial t}\right)$$

which is what we sought to show. QED.

How to Derive Maxwell's Curl E Equation

We want to show that Maxwell's rule for magnetic induction of electric fields by changing magnetic fields

$$\nabla \times E = \left(-\frac{\partial B}{\partial t}\right)$$

is a result of Coulomb's Law and the relativistic transformations (A-1, A-2, and A-3). That is, one need not use the notion of the magnetic field B or Maxwell's equation as the origin of the induced E field.

Choose two charges Q_c and Q_m moving in reference frame 2 at the same velocity v with respect to frame 1. Q_c is at the origin and Q_m is at the point $(x_2, y_2, 0)$. Viewed by an observer in frame 2, the charges are affected only by each other's Coulomb forces of attraction and repulsion; There are no magnetic fields present.

When viewed from frame 1 which can be our laboratory, one of the moving charges, say Q_c, is classically regarded as a current which produces a magnetic field B. The other charge Q_m will be affected by the magnetic field of moving Q_c according to the usual magnetic force rule,

$$F = Q_m(v \times B)$$

We want to get this result without using Maxwell's equation, by determining that the magnetic field B can be obtained from Coulombs law and the relativistic transformations.

Begin by writing the forces on Q_m due to Q_c using Coulomb's law as observed in frame 2,

$$F_2 = \frac{Q_m Q_c}{4\pi r_2^3}\left(x_2 + y_2\right)$$

where x_2 and y_2 are the vectors of position. Then, transform the components of this force to frame 1, using force transformations (A-4),

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$$F_{x1} = F_{x2} = \frac{Q_c Q_m x_2}{4\pi\varepsilon_0 \left(x_2^2 + y_2^2\right)^{3/2}}$$

$$F_{y1} = F_{y2} = \frac{Q_c Q_m y_2}{\gamma 4\pi\varepsilon_0 \left(x_2^2 + y_2^2\right)^{3/2}}$$

and

$$F_{z1} = 0$$

We must also transform the lengths x_2 and y_2 into frame 1 at $t_1 = 0$ using the Lorentz transformations (A-5). Then, rearranging, and bringing $(\gamma)^{-2} = 1 - v^2/c^2$ into the numerator, one gets,

$$F_{y1} = \frac{\gamma Q_c Q_m y_1}{4\pi\varepsilon_0 \left(\gamma^2 x_1^2 + y_1^2\right)^{3/2}} \left(1 - \frac{v^2}{c^2}\right)$$

and

$$F_{x1} = \frac{\gamma Q_c Q_m x_1}{4\pi\varepsilon_0 \left(\gamma^2 x_1^2 + y_1^2\right)^{3/2}}$$

These two terms can be combined into a term with the force pointed along the radial vector $r_1 = x_1 + y_1$, between charges, plus a vector product term with the velocity, pointed in the z direction,

$$F_1 = Q_m \left[\frac{\gamma Q_c r_1}{4\pi\varepsilon_0 \left(\gamma^2 x_1^2 + y_1^2\right)^{3/2}} \right] + Q_m \left[v \frac{\gamma Q_c v y_1 k}{4\pi\varepsilon_0 c^2 \left(\gamma^2 x_1^2 + y_1^2\right)^{3/2}} \right]$$

The first term is the ordinary Coulomb force between the charges, and the second term may be recognized as the magnetic force = Q (v x B). The magnetic field B is the result of the apparent current $(Q_C)v$ arising from the relative velocity v of the observer and the charge Q_C. The constants in the denominator are identified as the inverse of the usual magnetic force constant $\mu_O = 1/4\pi\varepsilon_O c^2$.

We have obtained the magnetic force law from Coulomb's law and relativity which is what we sought to show.

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