AB Quantum Module

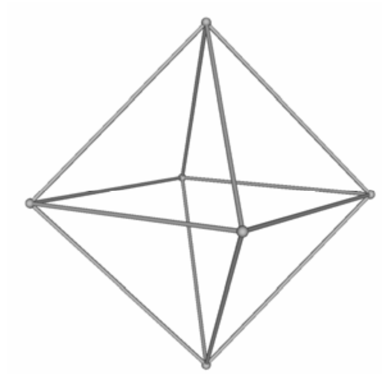

Figure 1 Regular Octahedron.

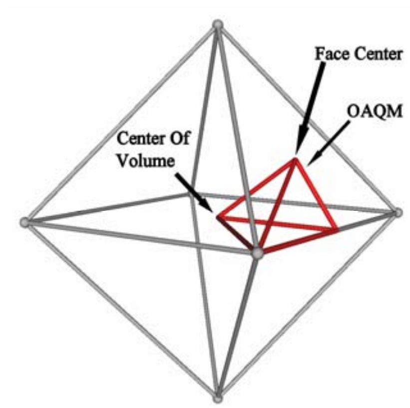

Figure 2 One of 48 Octahedron's AB Quantum Modules (red).

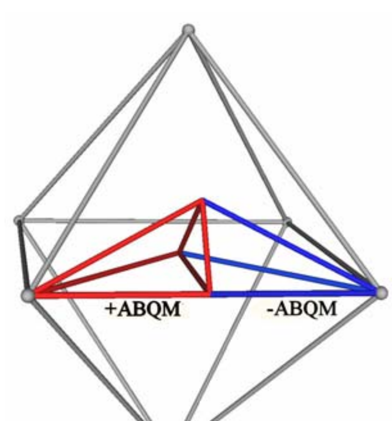

Figure 3 One +ABQM (red) and one –ABQM (blue).

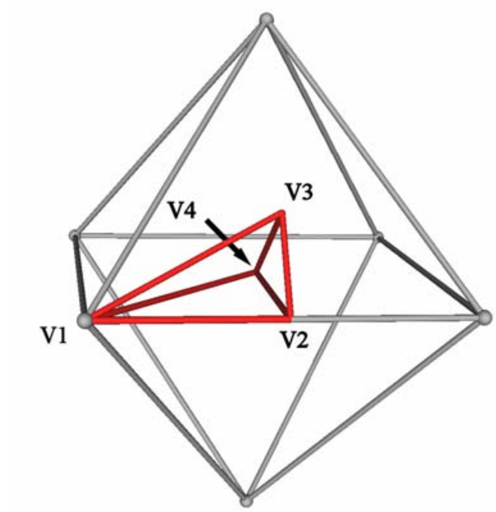

Figure 4 Vertex labeling for the ABQM.

Topology:

Vertices = 4

Edges = 6

Faces = 4 unequal triangles

Lengths:

EL ≡ Edge length of both the regular Tetrahedron and Octahedron.

V1 ≡ Vertex of regular Octahedron.

V2 ≡ Mid-edge of regular Octahedron.

V3 ≡ Face center of regular Octahedron.

V4 ≡ Center of volume of regular Octahedron.

Edge Lengths:

$$V1.V2 = \frac{1}{2}\ EL \equiv \text{Half of the regular Octahedron's edge length.}$$

$$V1.V3 = \frac{1}{\sqrt{3}}\ EL \cong 0.577\ 350\ 269\ EL = DFV_{Octa}$$

$$V1.V4 = \frac{1}{\sqrt{2}}\ EL \cong 0.707\ 106\ 781\ EL = DVV_{Octa}$$

$$V2.V3 = \frac{1}{2\sqrt{3}}\ EL \cong 0.288\ 675\ 135\ EL = DFE_{Octa}$$

$$V2.V4 = \frac{1}{2}\ EL \cong 0.5\ EL = DVE_{Octa}$$

$$V3.V4 = \frac{1}{\sqrt{6}}\ EL \cong 0.408\ 248\ 290\ 4\ EL = DVF_{Octa}$$

Center of Face to Vertex:

$$DF(V1.V2.V3)V(V1) = \frac{\sqrt{13}}{6\sqrt{3}} \text{ EL} \cong 0.346\ 944\ 333 \text{ EL}$$

$$DF(V1.V2.V3)V(V2) = \frac{1}{3\sqrt{3}} \text{ EL} \cong 0.192\ 450\ 090 \text{ EL}$$

$$DF(V1.V2.V3)V(V3) = \frac{\sqrt{7}}{6\sqrt{3}} \text{ EL} \cong 0.254\ 587\ 539 \text{ EL}$$

$$DF(V1.V2.V4)V(V1) = \frac{\sqrt{5}}{6} \text{ EL} \cong 0.372\ 677\ 996 \text{ EL}$$

$$DF(V1.V2.V4)V(V2) = \frac{\sqrt{2}}{6} \text{ EL} \cong 0.235\ 702\ 260 \text{ EL}$$

$$DF(V1.V2.V4)V(V4) = \frac{\sqrt{5}}{6} \text{ EL} \cong 0.372\ 677\ 996 \text{ EL}$$

$$DF(V1.V3.V4)V(V1) = \frac{1}{\sqrt{6}} \text{ EL} \cong 0.408\ 248\ 290\ 4 \text{ EL}$$

$$DF(V1.V3.V4)V(V3) = \frac{\sqrt{2}}{6} \text{ EL} \cong 0.235\ 702\ 260 \text{ EL}$$

$$DF(V1.V3.V4)V(V4) = \frac{1}{3} \text{ EL} \cong 0.333\ 333\ 333 \text{ EL}$$

$$DF(V2.V3.V4)V(V2) = \frac{\sqrt{2}}{6} \text{ EL} \cong 0.235\ 702\ 260 \text{ EL}$$

$$DF(V2.V3.V4)V(V3) = \frac{1}{6} \text{ EL} \cong 0.166\ 666\ 667 \text{ EL}$$

$$DF(V2.V3.V4)V(V4) = \frac{1}{2\sqrt{3}} \text{ EL} \cong 0.288\ 675\ 135 \text{ EL}$$

<u>Center of Face to Mid-edge:</u>

$$DF(V1.V2.V3)E(V1.V2) = \frac{\sqrt{7}}{12\sqrt{3}} \text{ EL} \cong 0.127\ 293\ 769 \text{ EL}$$

$$DF(V1.V2.V3)E(V1.V3) = \frac{1}{6\sqrt{3}} \text{ EL} \cong 0.096\ 225\ 045 \text{ EL}$$

$$DF(V1.V2.V3)E(V2.V3) = \frac{\sqrt{13}}{12\sqrt{3}} \text{ EL} \cong 0.173\ 472\ 167 \text{ EL}$$

$$DF(V1.V2.V4)E(V1.V2) = \frac{\sqrt{5}}{12} \text{ EL} \cong 0.186\ 338\ 998 \text{ EL}$$

$$DF(V1.V2.V4)E(V1.V4) = \frac{1}{6\sqrt{2}} \text{ EL} \cong 0.117\ 851\ 130 \text{ EL}$$

$$DF(V1.V2.V4)E(V2.V4) = \frac{\sqrt{5}}{12} \text{ EL} \cong 0.186\ 338\ 998 \text{ EL}$$

$$DF(V1.V3.V4)E(V1.V3) = \frac{1}{6} \text{ EL} \cong 0.166\,666\,667 \text{ EL}$$

$$DF(V1.V3.V4)E(V1.V4) = \frac{1}{6\sqrt{2}} \text{ EL} \cong 0.117\,851\,130 \text{ EL}$$

$$DF(V1.V3.V4)E(V3.V4) = \frac{1}{2\sqrt{6}} \text{ EL} \cong 0.204\,124\,145 \text{ EL}$$

$$DF(V2.V3.V4)E(V2.V3) = \frac{\sqrt{3}}{12} \text{ EL} \cong 0.144\,337\,567 \text{ EL}$$

$$DF(V2.V3.V4)E(V2.V4) = \frac{1}{12} \text{ EL} \cong 0.083\,333\,333 \text{ EL}$$

$$DF(V2.V3.V4)E(V3.V4) = \frac{1}{6\sqrt{2}} \text{ EL} \cong 0.117\,851\,130 \text{ EL}$$

<u>Center of Volume to Vertex:</u>

$$DVV(V1) = \frac{\sqrt{11}}{8} \text{ EL} \cong 0.414\,578\,098 \text{ EL}$$

$$DVV(V2) = \frac{\sqrt{3}}{8} \text{ EL} \cong 0.216\,506\,35 \text{ EL}$$

$$DVV(V3) = \frac{\sqrt{3}}{8} \text{ EL} \cong 0.216\,506\,35 \text{ EL}$$

$$DVV(V4) = \frac{5\sqrt{3}}{24} \text{ EL} \cong 0.360\,843\,918 \text{ EL}$$

Center of Volume to Mid-edge:

$$\text{DVE(V1.V2)} = \frac{\sqrt{3}}{8}\ \text{EL} \cong 0.216\ 506\ 35\ \text{EL}$$

$$\text{DVE(V1.V3)} = \frac{\sqrt{15}}{24}\ \text{EL} \cong 0.161\ 374\ 306\ \text{EL}$$

$$\text{DVE(V1.V4)} = \frac{\sqrt{15}}{24}\ \text{EL} \cong 0.161\ 374\ 306\ \text{EL}$$

$$\text{DVE(V2.V3)} = \frac{\sqrt{15}}{24}\ \text{EL} \cong 0.161\ 374\ 306\ \text{EL}$$

$$\text{DVE(V2.V4)} = \frac{\sqrt{15}}{24}\ \text{EL} \cong 0.161\ 374\ 306\ \text{EL}$$

$$\text{DVE(V3.V4)} = \frac{\sqrt{3}}{8}\ \text{EL} \cong 0.216\ 506\ 35\ \text{EL}$$

Center of Volume to Face Center:

$$\text{DVF(V1.V2.V3)} = \frac{5\sqrt{3}}{72}\ \text{EL} \cong 0.120\ 281\ 306\ \text{EL}$$

$$\text{DVF(V1.V2.V4)} = \frac{\sqrt{3}}{24}\ \text{EL} \cong 0.072\ 168\ 783\ \text{EL}$$

$$\text{DVF(V1.V3.V4)} = \frac{\sqrt{3}}{24}\ \text{EL} \cong 0.072\ 168\ 783\ \text{EL}$$

$$\text{DVF(V2.V3.V4)} = \frac{3\sqrt{11}}{72}\ \text{EL} \cong 0.138\ 192\ 699\ \text{EL}$$

Areas:

$$V1.V2.V3 = \frac{1}{8\sqrt{3}}\ EL^2 \cong 0.072\ 168\ 784\ EL^2$$

$$V1.V2.V4 = \frac{1}{8}\ EL^2 = 0.125\ EL^2$$

$$V1.V3.V4 = \frac{1}{6\sqrt{2}}\ EL^2 \cong 0.117\ 851\ 13\ EL^2$$

$$V2.V3.V4 = \frac{1}{12\sqrt{2}}\ EL^2 \cong 0.058\ 925\ 565\ EL^2$$

$$\text{Total face area} = \frac{3\sqrt{2} + 6\sqrt{3} + 3\sqrt{6}}{24\sqrt{6}}\ EL^2 \cong 0.373\ 945\ 478\ EL^2$$

Volume:

$$\text{Cubic measure volume equation} = \frac{\sqrt{2}}{144}\ EL^3 \cong 0.009\ 820\ 928\ EL^3.$$

$$\text{Synergetics' Tetra-volume equation} = \frac{1}{12}\ EL^3 \cong 0.083\ 333\ 333\ EL^3$$

Angles:

Face Angles:

Sum of face angles = 720°

Face V1.V2.V3:

V2.V1.V3 = 30°

V1.V2.V3 = 90°

V1.V3.V2 = 60°

Face V1.V2.V4:

V2.V1.V4 = 45.0°

V1.V2.V4 = 90°

V1.V4.V2 = 45.0°

Face V1.V3.V4:

$$V3.V1.V4 = \arcsin\left(\frac{1}{\sqrt{3}}\right) \cong 35.264\,389\,683°$$

V1.V3.V4 = 90°

$$V1.V4.V3 = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\,610\,317°$$

Face V2.V3.V4:

$$V3.V2.V4 = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610\ 317°$$

$$V2.V3.V4 = 90°$$

$$V2.V4.V3 = \arcsin\left(\frac{1}{\sqrt{3}}\right) \cong 35.264\ 389\ 683°$$

Central Angles (identified by edge labels):

$$V1.V2 = \arccos\left(\frac{-1}{\sqrt{33}}\right) \cong 100.024\ 988°$$

$$V1.V3 = \arccos\left(\frac{-11}{3\sqrt{33}}\right) \cong 129.664\ 035\ 4°$$

$$V1.V4 = \arccos\left(\frac{-19}{5\sqrt{33}}\right) \cong 131.413\ 997°$$

$$V2.V3 = \arccos\left(\frac{1}{9}\right) \cong 83.620\ 629\ 79°$$

$$V2.V4 = \arccos\left(\frac{-7}{15}\right) \cong 117.818\ 139\ 3°$$

$$V3.V4 = \arccos\left(\frac{1}{15}\right) \cong 86.177\ 446\ 27°$$

Dihedral Angles (identified by edge labels):

$$\text{V1.V2} = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\,610\,317°$$

V1.V3 = 90°

V1.V4 = 45°

V2.V3 = 90°

V2.V4 = 90°

V3.V4 = 60°

Vertex Coordinates (X, Y, Z):

$$\text{V1} = \left(\frac{1}{6}, \frac{-3}{8}, \frac{-1}{12\sqrt{2}}\right) \text{EL}$$

$\cong$ (0.166 666 667, -0.375, -0.058 925 565) EL

$$\text{V2} = \left(\frac{1}{6}, \frac{1}{8}, \frac{-1}{12\sqrt{2}}\right) \text{EL}$$

$\cong$ (0.166 666 667, 0.125, -0.058 925 565) EL

$$\text{V3} = \left(0, \frac{1}{8}, \frac{3}{12\sqrt{2}}\right) \text{EL}$$

$\cong$ (0.0, 0.125, 0.176 776 695) EL

$$\text{V4} = \left(\frac{-1}{3}, \frac{1}{8}, \frac{-1}{12\sqrt{2}}\right) \text{EL}$$

$\cong$ (-0.333 333 333, 0.125, -0.058 925 565) EL

Unfolded Vertex Coordinates (X, Y):

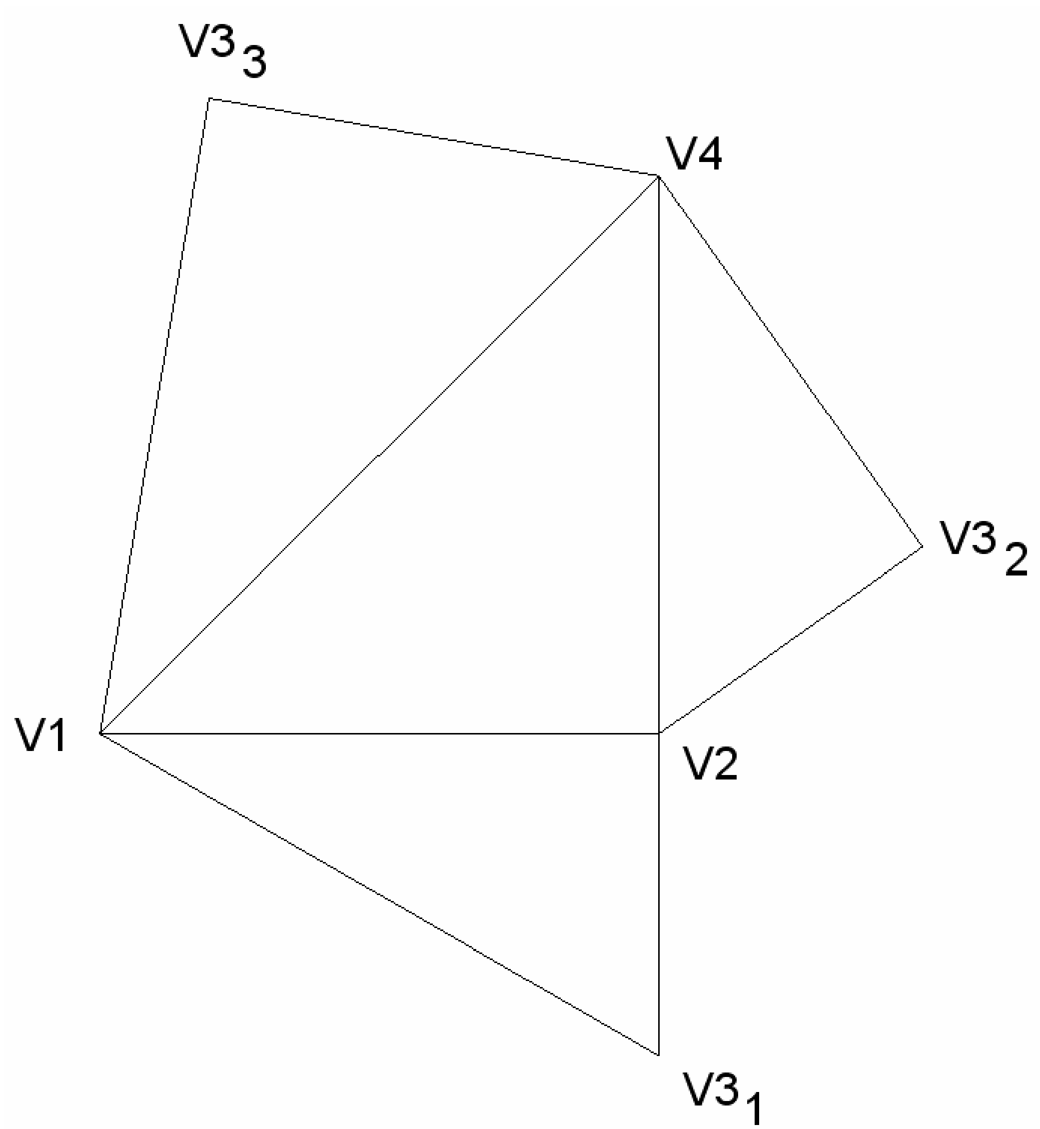

Figure 5 Layout for the Octahedron's AB Quantum Module.

V1 = (0.0, 0.0) EL

$$V2 = \left(\frac{1}{2}, 0.0\right) \text{ EL} \cong (0.5,\ 0.0) \text{ EL}$$

$$V3_1 = \left(\frac{1}{2}, \frac{-1}{2\sqrt{3}}\right) \text{EL} \cong (0.5,\ -0.288\ 675\ 134) \text{ EL}$$

$$V3_2 = \left(\frac{1}{2} + \frac{1}{3\sqrt{2}}, \frac{1}{6}\right) \text{ EL} \cong (0.735\ 702\ 26,\ 0.166\ 666\ 667) \text{ EL}$$

$$V3_3 = \left(\frac{\sqrt{2}-1}{3\sqrt{2}}, \frac{\sqrt{2}+1}{3\sqrt{2}}\right) \text{ EL} \cong (0.097\ 631\ 07,\ 0.569\ 035\ 59) \text{ EL}$$

$$V4 = \left(\frac{1}{2}, \frac{1}{2}\right) \text{ EL} \cong (0.5,\ 0.5) \text{ EL}$$

Comments:

As with the Tetrahedron's A Quantum Module, there are 2 polarizations (mirror images) of the Octahedron's AB Quantum module which we can label as +ABQM and –ABQM. See Figure 3 above.

The Octahedron's AB Quantum Module (ABQM) is composed of one A Quantum Module and one B Quantum Module.

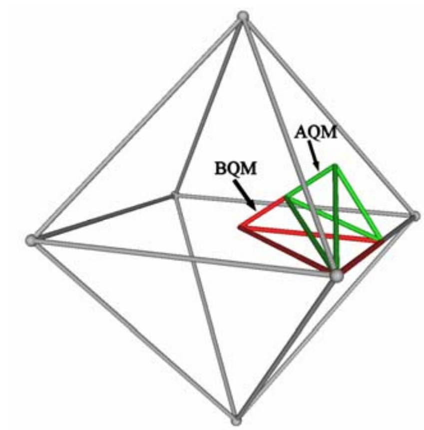

Figure 6 A (green) and B (red) Quantum Modules
define the Octahedron's AB Quantum Module.

In looking at the Quantum Module subdivision of polyhedra, it is often the case that a B Quantum model is face bond to (paired with) an A Quantum model.

The B Quantum Module does not follow all the symmetry lines and planes of the Octahedron. This may be why it is paired with an A Quantum module in the polyhedra which are capable of being subdivided into A and B Quantum Modules. The A Quantum Module paired with the B Quantum Module completes the Octahedron's line and plane symmetries.

Additionally, while the A Quantum module can be built out of half-sized A and B Quantum Modules, the B Quantum model can not be built out of smaller A and B Quantum Modules.

Therefore, it is more convenient to define and use the Octahedron's "AB" Quantum Module (ABQM) then it is to define and use the B Quantum Module when subdividing polyhedra into Quantum Modules.

The ABQM can be subdivided into smaller A Quantum Modules (AQM) and "AB" Quantum Modules (ABQM). Likewise, the AQM can be subdivided into smaller AQM and ABQM.