B Quantum Module

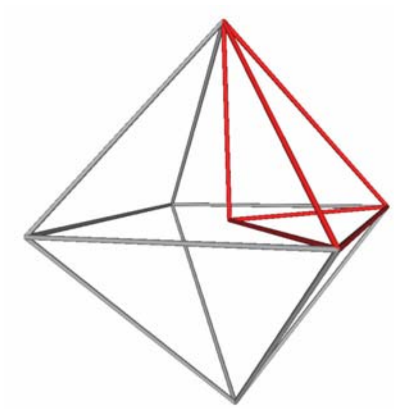

Figure 1 1/8th Octahedron (red) in the regular Octahedron.

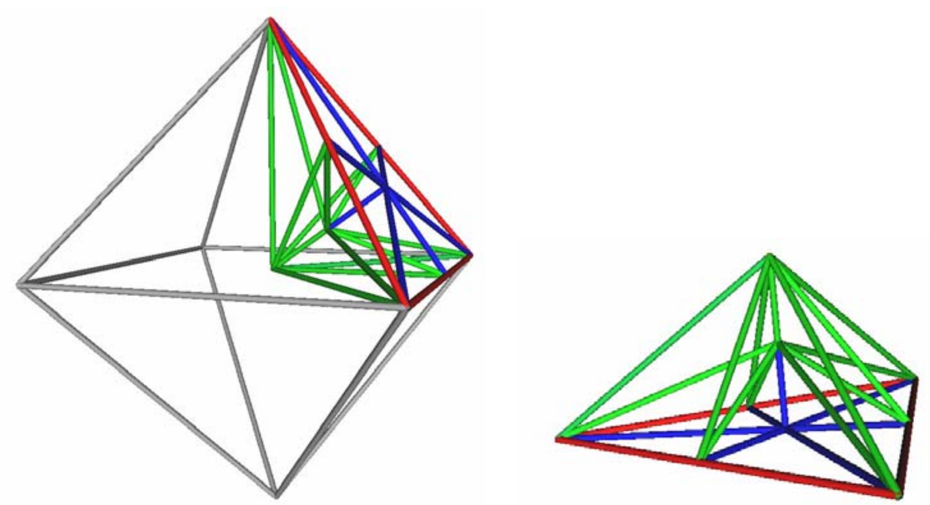

Figure 2 Subdivisions of the 1/8th Octahedron into
6 A (blue) and 6 B (green) Quantum Modules.

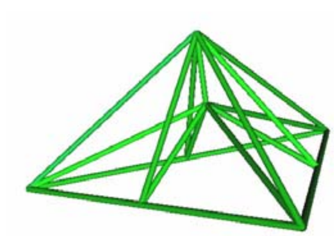

Figure 3 Collection of 6 (3+ and 3–) B Quantum Modules.

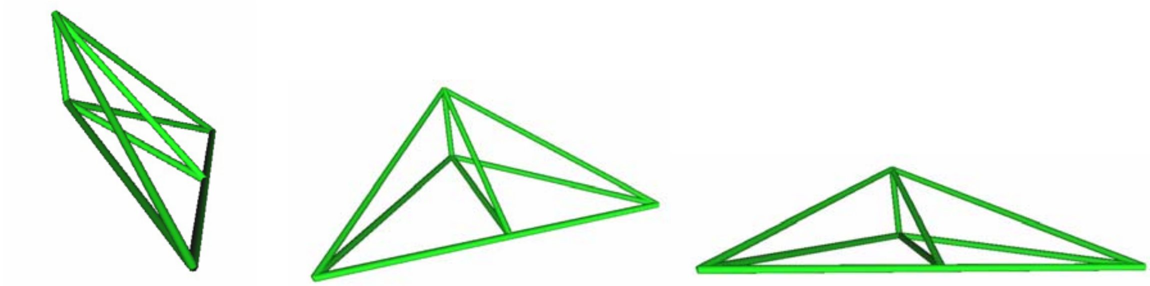

Figure 4 Different orientations of a positive and negative B Quantum Module pair.

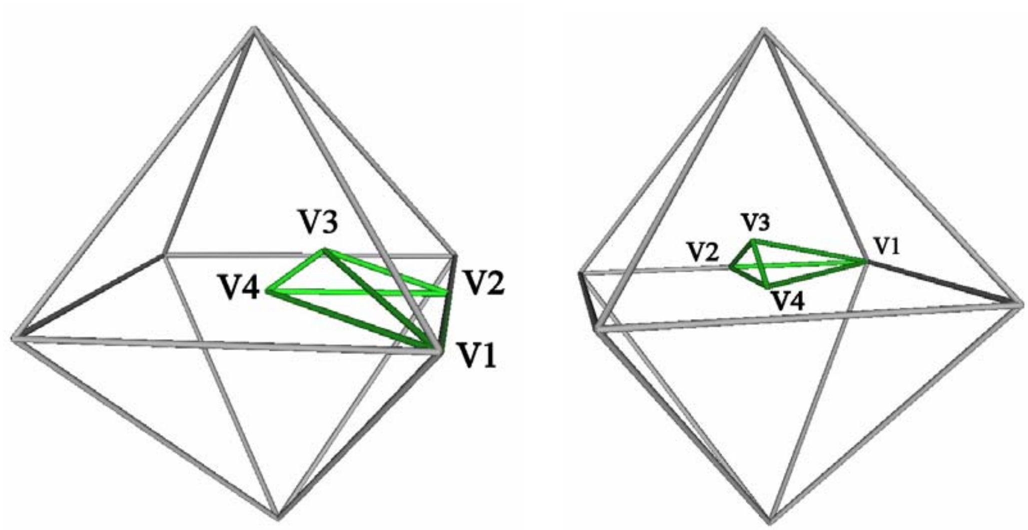

Figure 5 Single B Quantum Module within regular Octahedron.

Topology:

Vertices = 4

Edges = 6

Faces = 4 unequal triangles

Lengths:

EL ≡ Regular Tetrahedron edge length = Regular Octahedron edge length.

V1 ≡ Octahedron vertex.
V2 ≡ Octahedron mid-edge point.
V3 ≡ Half way from Octahedron center of volume to Octahedron face center.
V4 ≡ Octahedron center of volume.

Edge Lengths:

$$V1.V2 = \frac{1}{2}\ EL$$

$$V1.V3 = \frac{3}{2\sqrt{6}}\ EL \cong 0.612\ 372\ 436\ EL = DVV_{TETRAHEDRON}$$

$$V1.V4 = \frac{1}{\sqrt{2}}\ EL \cong 0.707\ 106\ 781\ EL = DVV_{OCTAHEDRON}$$

$$V2.V3 = \frac{1}{2\sqrt{2}}\ EL \cong 0.353\ 553\ 391\ EL = DVE_{TETRAHEDRON}$$

$$V2.V4 = \frac{1}{2}\ EL = 0.5\ EL = DVE_{OCTAHEDRON}$$

$$V3.V4 = \frac{1}{2\sqrt{6}}\ EL \cong 0.204\ 124\ 145\ EL = DVF_{TETRAHEDRON}$$

Center of Face to Vertex:

$$DF(V1.V2.V3)V(V1) = \frac{1}{2\sqrt{2}} \text{ EL} \cong 0.353\ 553\ 391 \text{ EL}$$

$$DF(V1.V2.V3)V(V2) = \frac{1}{2\sqrt{6}} \text{ EL} \cong 0.204\ 124\ 145 \text{ EL}$$

$$DF(V1.V2.V3)V(V3) = \frac{1}{2\sqrt{3}} \text{ EL} \cong 0.288\ 675\ 135 \text{ EL}$$

$$DF(V1.V2.V4)V(V1) = \frac{\sqrt{5}}{6} \text{ EL} \cong 0.372\ 677\ 996 \text{ EL}$$

$$DF(V1.V2.V4)V(V2) = \frac{\sqrt{2}}{6} \text{ EL} \cong 0.235\ 702\ 260 \text{ EL}$$

$$DF(V1.V2.V4)V(V4) = \frac{\sqrt{5}}{6} \text{ EL} \cong 0.372\ 677\ 996 \text{ EL}$$

$$DF(V1.V3.V4)V(V1) = \frac{\sqrt{41}}{6\sqrt{6}} \text{ EL} \cong 0.435\ 677\ 421 \text{ EL}$$

$$DF(V1.V3.V4)V(V3) = \frac{1}{3\sqrt{3}} \text{ EL} \cong 0.192\ 450\ 090 \text{ EL}$$

$$DF(V1.V3.V4)V(V4) = \frac{\sqrt{17}}{6\sqrt{6}} \text{ EL} \cong 0.280\ 541\ 804 \text{ EL}$$

$$\mathrm{DF(V2.V3.V4)V(V2)} = \frac{\sqrt{17}}{6\sqrt{6}}\ \mathrm{EL} \cong 0.280\ 541\ 804\ \mathrm{EL}$$

$$\mathrm{DF(V2.V3.V4)V(V3)} = \frac{1}{6\sqrt{3}}\ \mathrm{EL} \cong 0.096\ 225\ 045\ \mathrm{EL}$$

$$\mathrm{DF(V2.V3.V4)V(V4)} = \frac{\sqrt{11}}{6\sqrt{6}}\ \mathrm{EL} \cong 0.225\ 667\ 733\ \mathrm{EL}$$

<u>Center of Face to Mid-edge:</u>

$$\mathrm{DF(V1.V2.V3)E(V1.V2)} = \frac{1}{4\sqrt{3}}\ \mathrm{EL} \cong 0.144\ 337\ 567\ \mathrm{EL}$$

$$\mathrm{DF(V1.V2.V3)E(V1.V3)} = \frac{1}{4\sqrt{6}}\ \mathrm{EL} \cong 0.102\ 062\ 073\ \mathrm{EL}$$

$$\mathrm{DF(V1.V2.V3)E(V2.V3)} = \frac{1}{4\sqrt{2}}\ \mathrm{EL} \cong 0.176\ 776\ 695\ \mathrm{EL}$$

$$\mathrm{DF(V1.V2.V4)E(V1.V2)} = \frac{\sqrt{5}}{12}\ \mathrm{EL} \cong 0.186\ 338\ 998\ \mathrm{EL}$$

$$\mathrm{DF(V1.V2.V4)E(V1.V4)} = \frac{1}{6\sqrt{2}}\ \mathrm{EL} \cong 0.117\ 851\ 130\ \mathrm{EL}$$

$$\mathrm{DF(V1.V2.V4)E(V2.V4)} = \frac{\sqrt{5}}{12}\ \mathrm{EL} \cong 0.186\ 338\ 998\ \mathrm{EL}$$

$$\mathrm{DF(V1.V3.V4)E(V1.V3)} = \frac{\sqrt{17}}{12\sqrt{6}} \text{ EL} \cong 0.140\ 270\ 902 \text{ EL}$$

$$\mathrm{DF(V1.V3.V4)E(V1.V4)} = \frac{1}{6\sqrt{3}} \text{ EL} \cong 0.096\ 225\ 045 \text{ EL}$$

$$\mathrm{DF(V1.V3.V4)E(V3.V4)} = \frac{\sqrt{41}}{12\sqrt{6}} \text{ EL} \cong 0.217\ 838\ 710 \text{ EL}$$

$$\mathrm{DF(V2.V3.V4)E(V2.V3)} = \frac{\sqrt{11}}{12\sqrt{6}} \text{ EL} \cong 0.112\ 833\ 867 \text{ EL}$$

$$\mathrm{DF(V2.V3.V4)E(V2.V4)} = \frac{1}{12\sqrt{3}} \text{ EL} \cong 0.048\ 112\ 522 \text{ EL}$$

$$\mathrm{DF(V2.V3.V4)E(V3.V4)} = \frac{\sqrt{17}}{12\sqrt{6}} \text{ EL} \cong 0.140\ 270\ 902 \text{ EL}$$

<u>Center of Volume to Vertex:</u>

$$\mathrm{DVV(V1)} = \frac{\sqrt{71}}{8\sqrt{6}} \text{ EL} \cong 0.429\ 995\ 155 \text{ EL}$$

$$\mathrm{DVV(V2)} = \frac{\sqrt{23}}{8\sqrt{6}} \text{ EL} \cong 0.244\ 736\ 253 \text{ EL}$$

$$\mathrm{DVV(V3)} = \frac{\sqrt{5}}{8\sqrt{2}} \text{ EL} \cong 0.197\ 642\ 354 \text{ EL}$$

$$\mathrm{DVV(V4)} = \frac{\sqrt{13}}{8\sqrt{2}} \text{ EL} \cong 0.244\ 736\ 253 \text{ EL}$$

Center of Volume to Mid-edge:

$$\text{DVE(V1.V2)} = \frac{\sqrt{23}}{8\sqrt{6}} \text{ EL} \cong 0.244\ 736\ 253 \text{ EL}$$

$$\text{DVE(V1.V3)} = \frac{\sqrt{7}}{8\sqrt{6}} \text{ EL} \cong 0.135\ 015\ 431 \text{ EL}$$

$$\text{DVE(V1.V4)} = \frac{\sqrt{7}}{8\sqrt{6}} \text{ EL} \cong 0.135\ 015\ 431 \text{ EL}$$

$$\text{DVE(V2.V3)} = \frac{\sqrt{7}}{8\sqrt{6}} \text{ EL} \cong 0.135\ 015\ 431 \text{ EL}$$

$$\text{DVE(V2.V4)} = \frac{\sqrt{7}}{8\sqrt{6}} \text{ EL} \cong 0.135\ 015\ 431 \text{ EL}$$

$$\text{DVE(V3.V4)} = \frac{\sqrt{23}}{8\sqrt{6}} \text{ EL} \cong 0.244\ 736\ 253 \text{ EL}$$

Center of Volume to Face Center:

$$\text{DVF(V1.V2.V3)} = \frac{\sqrt{13}}{24\sqrt{2}} \text{ EL} \cong 0.106\ 229\ 573 \text{ EL}$$

$$\text{DVF(V1.V2.V4)} = \frac{\sqrt{5}}{24\sqrt{2}} \text{ EL} \cong 0.065\ 880\ 785 \text{ EL}$$

$$\text{DVF(V1.V3.V4)} = \frac{\sqrt{23}}{24\sqrt{6}} \text{ EL} \cong 0.081\ 578\ 751 \text{ EL}$$

$$\text{DVF(V2.V3.V4)} = \frac{\sqrt{71}}{24\sqrt{6}} \text{ EL} \cong 0.143\ 331\ 718 \text{ EL}$$

Areas:

$$\text{V1.V2.V3} = \frac{1}{8\sqrt{2}}\ \text{EL}^2 \cong 0.088\ 388\ 348\ \text{EL}^2$$

$$\text{V1.V2.V4} = \frac{1}{8}\ \text{EL}^2 = 0.125\ \text{EL}^2$$

$$\text{V1.V3.V4} = \frac{1}{12\sqrt{2}}\ \text{EL}^2 \cong 0.058\ 925\ 565\ \text{EL}^2$$

$$\text{V2.V3.V4} = \frac{1}{24\sqrt{2}}\ \text{EL}^2 \cong 0.029\ 462\ 783\ \text{EL}^2$$

$$\text{Total face area} = \frac{1+\sqrt{2}}{8}\ \text{EL}^2 \cong 0.301\ 776\ 695\ \text{EL}^2$$

$$= \frac{1+\sqrt{2}}{2}\ (\text{V1.V2})^2 \cong 1.207\ 106\ 781\ (\text{V1.V2})^2$$

Volume:

$$\text{Cubic measure volume equation} = \frac{1}{144\sqrt{2}}\ \text{EL}^3 \cong 0.004\ 910\ 464\ \text{EL}^3.$$

$$\text{Synergetics' Tetra-volume equation} = \frac{1}{24}\ \text{EL}^3 \cong 0.041\ 666\ 667\ \text{EL}^3$$

Angles:

Face Angles:

Sum of face angles = 720°

Face V1.V2.V3:

$$\text{V2.V1.V3} = \arcsin\left(\frac{1}{\sqrt{3}}\right) \cong 35.264\ 389\ 683°$$

V1.V2.V3 = 90°

$$\text{V1.V3.V2} = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610\ 317°$$

Face V1.V2.V4:

V2.V1.V4 = 45°

V1.V2.V4 = 90°

V1.V4.V2 = 45°

Face V1.V3.V4:

$$\text{V3.V1.V4} = \arcsin\left(\frac{1}{\sqrt{3}}\right) - \arccos\left(\frac{2\sqrt{2}}{3}\right) \cong 15.793\ 169\ 048^\circ$$

$$\text{V1.V3.V4} = 90^\circ + \arccos\left(\frac{2\sqrt{2}}{3}\right) \cong 109.471\ 220\ 634^\circ$$

$$\text{V1.V4.V3} = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610\ 317^\circ$$

Face V2.V3.V4:

$$\text{V3.V2.V4} = \arccos\left(\frac{2\sqrt{2}}{3}\right) \cong 19.471\ 220\ 634^\circ$$

$$\text{V2.V3.V4} = 90^\circ + \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 125.264\ 389\ 683^\circ$$

$$\text{V2.V4.V3} = \arcsin\left(\frac{1}{\sqrt{3}}\right) \cong 35.264\ 389\ 683^\circ$$

Central Angles (identified by edge labels):

$$\text{V1.V2} = \arccos\left(\frac{-1}{\sqrt{1633}}\right) \cong 91.417\ 992\ 307^\circ$$

$$\text{V1.V3} = \arccos\left(\frac{-29\sqrt{12}}{6\sqrt{355}}\right) \cong 152.702\ 163\ 172^\circ$$

$$\text{V1.V4} = \arccos\left(\frac{-41\sqrt{12}}{6\sqrt{923}}\right) \cong 141.183\ 029\ 765^\circ$$

$$\text{V2.V3} = \arccos\left(\frac{-5\sqrt{12}}{6\sqrt{115}}\right) \cong 105.616\ 129\ 405°$$

$$\text{V2.V4} = \arccos\left(\frac{-17\sqrt{12}}{6\sqrt{299}}\right) \cong 124.583\ 973\ 480°$$

$$\text{V3.V4} = \arccos\left(\frac{19}{3\sqrt{65}}\right) \cong 38.228\ 117\ 494°$$

Dihedral Angles (identified by edge labels):

$$\text{V1.V2} = \arcsin\left(\frac{\sqrt{2}}{\sqrt{3}}\right) - \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 19.471\ 220\ 634°$$

V1.V3 = 120°

V1.V4 = 45°

V2.V3 = 90°

V2.V4 = 90°

V3.V4 = 60°

Vertex Coordinates (X, Y, Z):

$$V1 = \left(\frac{-3}{8}, \frac{-1}{4\sqrt{3}}, \frac{-3}{8\sqrt{6}} \right) EL$$

$$\cong (-0.375,\ -0.144\ 337\ 567,\ -0.153\ 093\ 109)\ EL$$

$$V2 = \left(\frac{1}{8}, \frac{-1}{4\sqrt{3}}, \frac{-3}{8\sqrt{6}} \right) EL$$

$$\cong (0.125,\ -0.144\ 337\ 567,\ -0.153\ 093\ 109)\ EL$$

$$V3 = \left(\frac{1}{8}, \frac{1}{4\sqrt{3}}, \frac{1}{8\sqrt{6}} \right) EL$$

$$\cong (0.125,\ 0.144\ 337\ 567,\ 0.051\ 031\ 036)\ EL$$

$$V4 = \left(\frac{1}{8}, \frac{1}{4\sqrt{3}}, \frac{5}{8\sqrt{6}} \right) EL$$

$$\cong (0.125,\ 0.144\ 337\ 567,\ 0.255\ 155\ 182)\ EL$$

Unfolded Vertex Coordinates (X, Y):

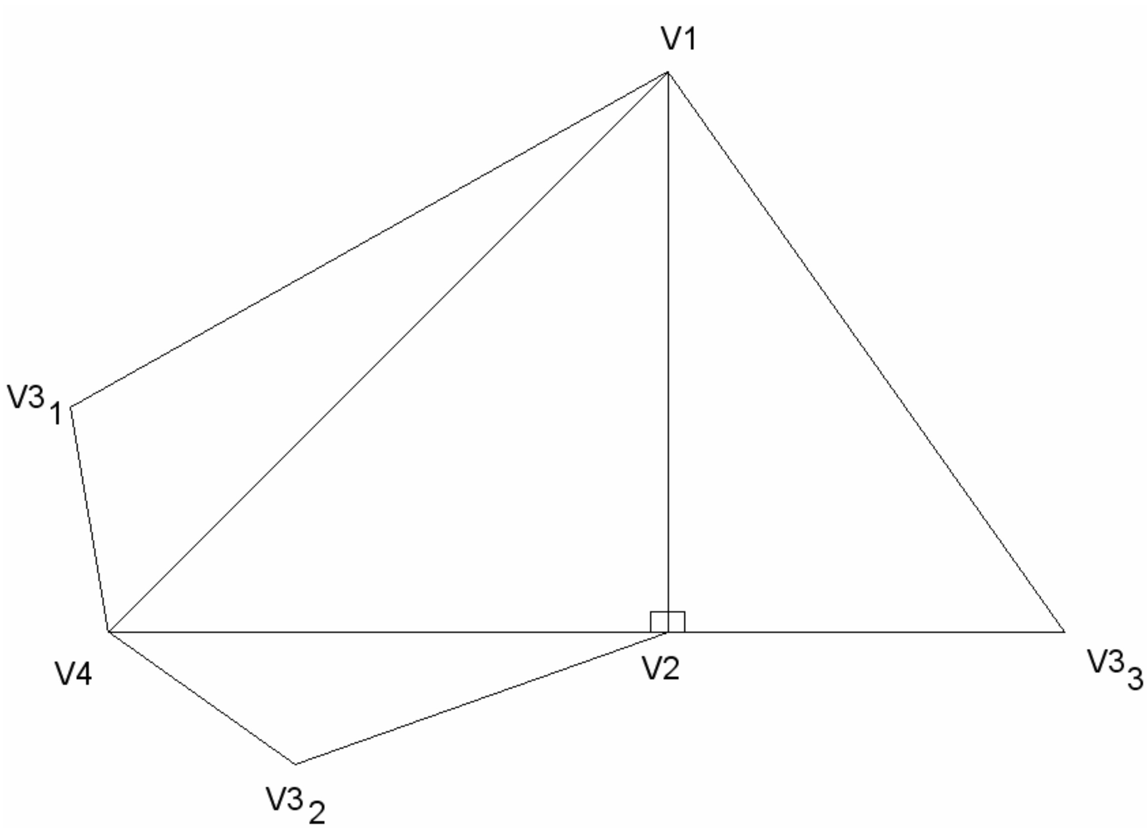

Figure 7 Layout for the B Quantum Module.

$$\alpha = \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 35.264\,389\,683°$$

$$\beta = 45^{\circ} + \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 99.735\,61°$$

V1 = (0.5, 0.5) EL

V2 = (0.5, 0.0) EL

$$V3_1 = \left(\frac{1}{2\sqrt{6}}\cos(\beta), \frac{1}{2\sqrt{6}}\sin(\beta)\right) \text{EL} \cong (-0.034\,51, 0.201\,185) \text{ EL}$$

$$V3_2 = \left(\frac{1}{2\sqrt{6}}\cos(\alpha), \frac{-1}{2\sqrt{6}}\sin(\alpha)\right) \text{EL} \cong (0.166\,667, -0.117\,851) \text{ EL}$$

$$V3_3 = \left(\frac{1+\sqrt{2}}{2\sqrt{2}}, 0.0\right) \text{EL} \cong (0.853\,553, 0.0) \text{ EL}$$

V4 = (0.0, 0.0) EL

Comments:

The B Quantum Module is also constructed from 1/8th Octahedron, which is a sub-polyhedron of the Octahedron.

There are 2 different B Quantum Modules labeled B+ and B-. These are mirror images of each other. The B+ Quantum Model can be opened and folded into the B- Quantum Model and visa versa.

The B Quantum Module does not fill all-space by itself.

The dual of the B Quantum Module is another (different) irregular Tetrahedron which is not considered further in this text.