Octahedron

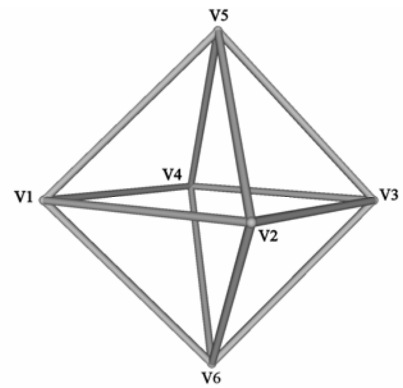

Figure 1 *Octahedron and vertex labeling.*

Topology:

Vertices = 6

Edges = 12

Faces = 8 equilateral triangles

Lengths:

EL ≡ Edge length of the Octahedron.

FA = $\frac{\sqrt{3}}{2}$ EL ≅ 0.866 025 403 7 EL = triangular face altitude.

DFE = $\frac{1}{2\sqrt{3}}$ EL ≅ 0.288 675 134 5 EL

DFV = $\frac{1}{\sqrt{3}}$ EL ≅ 0.577 350 269 1 EL

DVE = $\frac{1}{2}$ EL ≅ 0.5 EL

DVF = $\frac{1}{\sqrt{6}}$ EL ≅ 0.408 248 290 4 EL

DVV = $\frac{1}{\sqrt{2}}$ EL ≅ 0.707 106 781 1 EL

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Areas:

Each of the 8 equilateral triangular face have the same area.

Area of one triangular face = $\frac{\sqrt{3}}{4}\ EL^2 \cong 0.433\ 012\ 701\ 8\ \ EL^2$

Total face area = $2\sqrt{3}\ \ EL^2 \cong 3.464\ 101\ 615\ 1\ EL^2$

Volume:

Cubic measured volume equation = $\frac{\sqrt{2}}{3}\ EL^3 \cong 0.471\ 404\ 520\ 7\ EL^3$

Synergetics' Tetra-volume equation = $4\ EL^3$

Angles:

All face angles are 60°

Sum of face angles = 1440°

Central Angles:

All central angles are = 90°

Dihedral Angles:

All dihedral angles are = $2\arcsin\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 109.471\ 220\ 634\ 4°$

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Additional Angle Information:

Note that

Central Angle(Octahedron) + Dihedral Angle(Cube) = 180°

Central Angle(Cube) + Dihedral Angle(Octahedron) = 180°

which is the case for pairs of dual polyhedra.

When the Octahedron is spun about an axis through the center of opposite faces, a cone is defined. The apex of the cone is the Octahedron's center of volume. Three vertices of an Octahedron's triangular face lay on the surface of the cone.

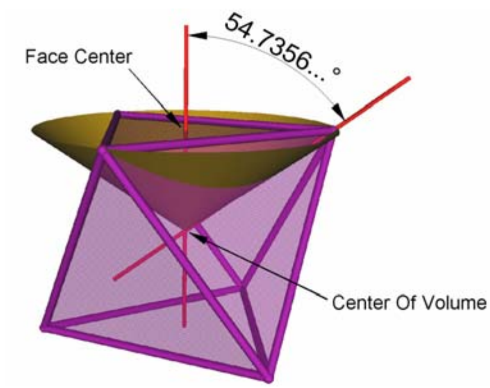

Figure 2 *Octahedron defined 1/2 Cone angle.*

The half-cone angle is

$$\arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610\ 317°$$

Vertex Coordinates (x, y, z):

$$V1 = \left(\frac{-1}{2}, \frac{-1}{2}, 0.0 \right) \text{EL}$$

$$\cong (-0.5, -0.5, 0.0) \text{ EL}$$

$$V2 = \left(\frac{1}{2}, \frac{-1}{2}, 0.0 \right) \text{EL}$$

$$\cong (0.5, -0.5, 0.0) \text{ EL}$$

$$V3 = \left(\frac{1}{2}, \frac{1}{2}, 0.0 \right) \text{EL}$$

$$\cong (0.5, 0.5, 0.0) \text{ EL}$$

$$V4 = \left(\frac{-1}{2}, \frac{1}{2}, 0.0 \right)$$

$$\cong (-0.5, 0.5, 0.0) \text{ EL}$$

$$V5 = \left(0.0, 0.0, \frac{1}{\sqrt{2}} \right)$$

$$\cong (0.0, 0.0, 0.707\ 106\ 781) \text{ EL}$$

$$V6 = \left(0.0, 0.0, \frac{-1}{\sqrt{2}} \right)$$

$$\cong (0.0, 0.0, -0.707\ 106\ 781) \text{ EL}$$

Edge Map:

{{V1, V2} {V1, V4} {V1, V5} {V1, V6} {V2, V3} {V2, V5} {V2, V6}
{V3, V4} {V3, V5} {V3, V6} {V4, V5} {V4, V6}}

Face Maps:

{{V1, V2, V5} {V1, V6, V2} {V1, V5, V4} {V1, V4, V6}
{V3, V5, V2} {V3, V2, V6} {V3, V4, V5} {V3, V6, V4}}

Other Orientations:

There are 5 Octahedra having the same 30 vertices as 30 out of 62 vertices of the "120 Polyhedron (Type III: Dennis)". The pattern of these 30 vertex coordinate numbers is rather interesting when written in terms of the Golden Mean

$\varphi = \dfrac{1+\sqrt{5}}{2}$. In this case, the edge lengths of the Octahedra are

EL = $2\sqrt{2}\,\varphi^2 \cong 7.404\,918\,348$ units of length.

Using the vertex labeling of the 120 Polyhedron (Type III: Dennis) the vertex coordinates are as follows.

The vertex labels are those used for the 120 Polyhedron.

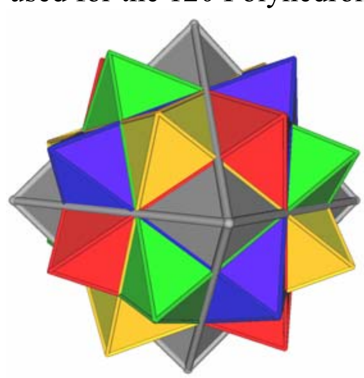

Figure 3 *Five intersecting Octahedra.*

Orientation 1:

V7 $= (-\varphi, -\varphi^2, \varphi^3) \cong (-1.618\,033\,989, -2.618\,033\,989, 4.236\,067\,977)$
V10 $= (\varphi^3, \varphi, \varphi^2) \cong (4.236\,067\,977, 1.618\,033\,989, 2.618\,033\,989)$
V22 $= (-\varphi^2, \varphi^3, \varphi) \cong (-2.618\,033\,989, 4.236\,067\,977, 1.618\,033\,989)$
V43 $= (\varphi^2, -\varphi^3, -\varphi) \cong (2.618\,033\,989, -4.236\,067\,977, -1.618\,033\,989)$
V49 $= (-\varphi^3, -\varphi, -\varphi^2) \cong (-4.236\,067\,977, -1.618\,033\,989, -2.618\,033\,989)$
V55 $= (\varphi, \varphi^2, -\varphi^3) \cong (1.618\,033\,989, 2.618\,033\,989, -4.236\,067\,977)$

Orientation 2:

V9 $= (\varphi, -\varphi^2, \varphi^3) \cong (1.618\,033\,989, -2.618\,033\,989, 4.236\,067\,977)$
V14 $= (-\varphi^3, \varphi, \varphi^2) \cong (-4.236\,067\,977, 1.618\,033\,989, 2.618\,033\,989)$
V21 $= (\varphi^2, \varphi^3, \varphi) \cong (2.618\,033\,989, 4.236\,067\,977, 1.618\,033\,989)$
V42 $= (-\varphi^2, -\varphi^3, -\varphi) \cong (-2.618\,033\,989, -4.236\,067\,977, -1.618\,033\,989)$
V53 $= (\varphi^3, -\varphi, -\varphi^2) \cong (4.236\,067\,977, -1.618\,033\,989, -2.618\,033\,989)$
V57 $= (-\varphi, \varphi^2, -\varphi^3) \cong (-1.618\,033\,989, 2.618\,033\,989, -4.236\,067\,977)$

Orientation 3:

V3 $= (\varphi, \varphi^2, \varphi^3) \cong (1.618\,033\,989, 2.618\,033\,989, 4.236\,067\,977)$
V15 $= (-\varphi^3, -\varphi, \varphi^2) \cong (-4.236\,067\,977, -1.618\,033\,989, 2.618\,033\,989)$
V25 $= (\varphi^2, -\varphi^3, \varphi) \cong (2.618\,033\,989, -4.236\,067\,977, 1.618\,033\,989)$
V40 $= (-\varphi^2, \varphi^3, -\varphi) \cong (-2.618\,033\,989, 4.236\,067\,977, -1.618\,033\,989)$
V44 $= (\varphi^3, \varphi, -\varphi^2) \cong (4.236\,067\,977, 1.618\,033\,989, -2.618\,033\,989)$
V59 $= (-\varphi, -\varphi^2, -\varphi^3) \cong (-1.618\,033\,989, -2.618\,033\,989, -4.236\,067\,977)$

Orientation 4:

V5 $= (-\varphi, \varphi^2, \varphi^3) \cong (-1.618\,033\,989, 2.618\,033\,989, 4.236\,067\,977)$
V19 $= (\varphi^3, -\varphi, \varphi^2) \cong (4.236\,067\,977, -1.618\,033\,989, 2.618\,033\,989)$
V24 $= (-\varphi^2, -\varphi^3, \varphi) \cong (-2.618\,033\,989, -4.236\,067\,977, 1.618\,033\,989)$
V39 $= (\varphi^2, \varphi^3, -\varphi) \cong (2.618\,033\,989, 4.236\,067\,977, -1.618\,033\,989)$
V48 $= (-\varphi^3, \varphi, -\varphi^2) \cong (-4.236\,067\,977, 1.618\,033\,989, -2.618\,033\,989)$
V61 $= (\varphi, -\varphi^2, -\varphi^3) \cong (1.618\,033\,989, -2.618\,033\,989, -4.236\,067\,977)$

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Orientation 5:

V1 = $(0, 0, 2\varphi^2) \cong (0.0, 0.0, 5.236\,067\,977)$
V26 = $(2\varphi^2, 0, 0) \cong (5.236\,067\,977, 0.0, 0.0)$
V29 = $(0, 2\varphi^2, 0) \cong (0.0, 5.236\,067\,977, 0.0)$
V32 = $(-2\varphi^2, 0, 0) \cong (-5.236\,067\,977, 0.0, 0.0)$
V35 = $(0, -2\varphi^2, 0) \cong (0.0, -5.236\,067\,977, 0.0)$
V62 = $(0, 0, -2\varphi^2) \cong (0.0, 0.0, -5.236\,067\,977)$

Unfolded Vertex Coordinates (X, Y):

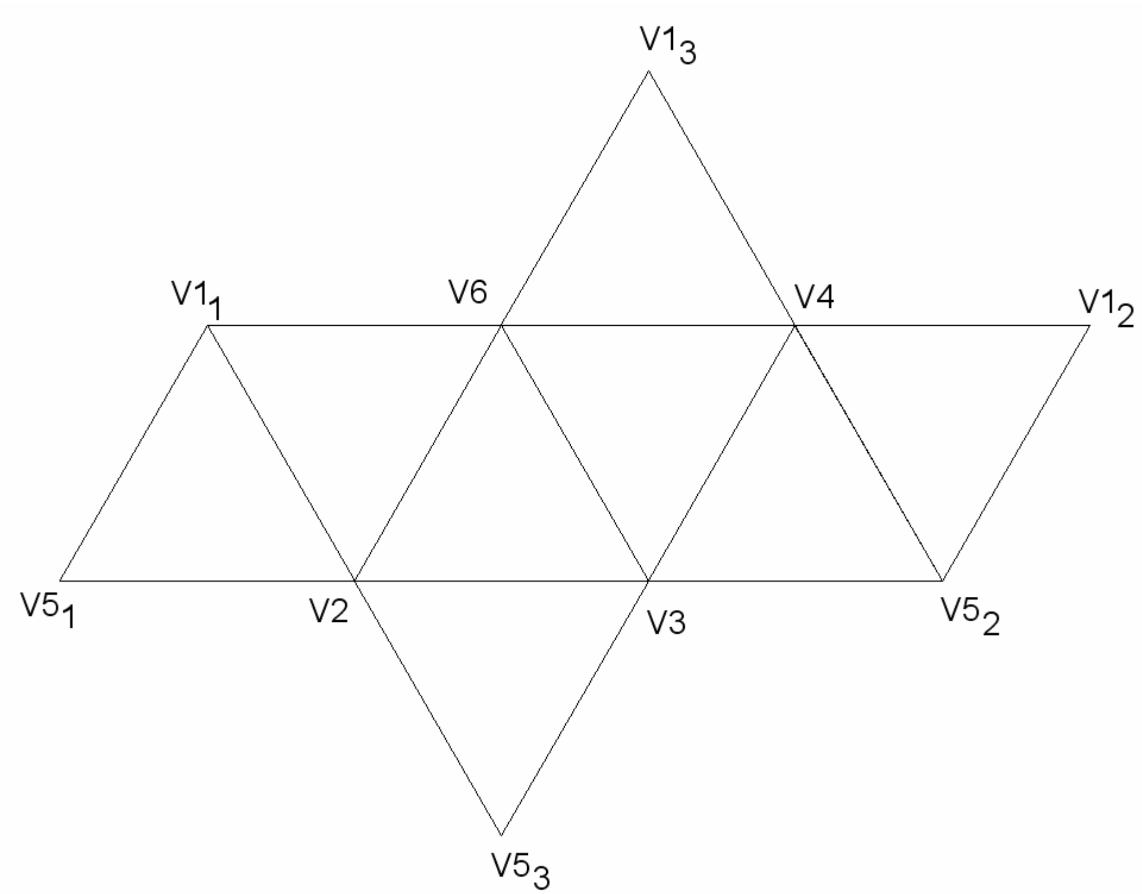

Figure 4 *Layout for the Octahedron.*

$$V1_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right) EL = (0.5, 0.866\ 025\ 403)\ EL$$

$$V1_2 = \left(\frac{7}{2}, \frac{\sqrt{3}}{2} \right) EL = (3.5, 0.866\ 025\ 403)\ EL$$

$$V1_3 = \left(2, \sqrt{3} \right) EL = (2.0, 1.732\ 050\ 808)\ EL$$

$$V2 = (1, 0)\ EL = (1.0, 0.0)\ EL$$

$$V3 = (2, 0)\ EL = (2.0, 0.0)\ EL$$

$$V4 = \left(\frac{5}{2}, \frac{\sqrt{3}}{2} \right) EL = (2.5, 0.866\ 025\ 403)\ EL$$

$V5_1 = (0, 0)$ EL = (0.0, 0.0) EL

$V5_2 = (3, 0)$ EL = (3.0, 0.0) EL

$$V5_3 = \left(\frac{3}{2}, \frac{-\sqrt{3}}{2} \right) EL = (1.5, -0.866\ 025\ 403)\ EL$$

$$V6 = \left(\frac{3}{2}, \frac{\sqrt{3}}{2} \right) EL = (2.5, 0.866\ 025\ 403)\ EL$$

Comments:

The Octahedron does not fill all-space by itself. It can be combined with the Tetrahedron to form an Octet which does fill all-space.

The dual of the Octahedron is the Cube.

The Octahedron is one of the polyhedra which the Jitterbug motion passes through.

By dividing the Octahedron's edges into Golden Ratio segments an Icosahedron is defined. Eight of the Icosahedron faces are in the same plane as the Octahedron's faces.

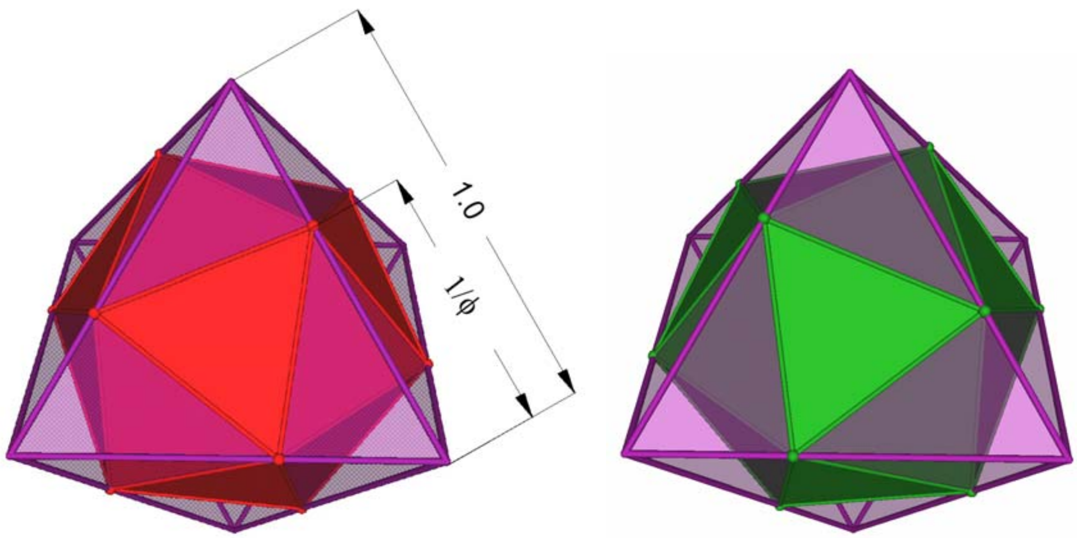

Figure 5 *Icosahedron defined within the Octahedron. There are two different orientations for the Icosahedron.*

The Skew Angle is the amount that a triangle in the Octahedron face needs to be rotated about the face center to match up with the orientation of the Icosahedron triangle.

$$\theta_{SKEW} = \pm \operatorname{acrtan}\left(\frac{(2-\varphi)\sqrt{3}}{\varphi} \right) \simeq \pm 22.238\,756^{\circ}$$

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Figure 6 *Defining the Skew Angle.*

The 5 intersecting Octahedra, as shown in Figure 3, can be formed by considering all 5 Octahedron to be initially coincident and then rotating 4 of the 5 Octahedra into position. The 4 rotation axes are the 4 Face-center-to-Face-center axes of the one fixed Octahedron (which also happens to be the 4 vertex-to-vertex axes of a Cube). The rotation angle is given by

$$\theta = \pm 2\ \text{acrsin}\left(\frac{\sqrt{3}}{2\sqrt{2}\ \varphi} \right) \simeq \pm 44.477\,512^{\circ}$$

which turns out to be twice the Skew Angle.