1/4-Tetrahedron

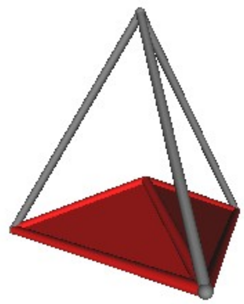

Figure 1 *Quarter Tetrahedron within regular Tetrahedron.*

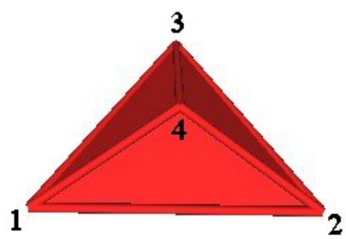

Figure 2 *Vertex labels.*

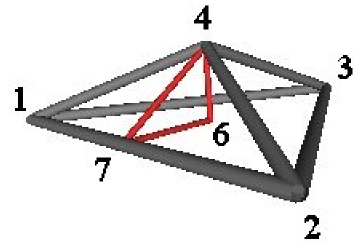

Figure 3 *Further point of interest.*

In this section, EL, DFE_T, DFV_T, DVE_T, DVF_T refer to the regular Tetrahedron, while DFE, DFV, DVE, DVF, DVV refer to the 1/4-Tetrahedron.

Since the (V1.V2.V4), (V1.V3.V4), (V2.V3.V4) faces are all the same, only data for the (V1.V2.V4) face needs to be calculated (along with the V1.V2.V3 face data.)

Topology:

Vertices = 4

Edges = 6

Faces = 4 triangles, 3 of which are the same.

Lengths:

EL ≡ Edge length of the regular Tetrahedron.

V1.V2 = V2.V3 = V1.V3 = EL

$$V1.V4 = V2.V4 = V3.V4 = \frac{3}{2\sqrt{6}}\ EL \cong 0.612\ 372\ 436\ EL = DVV_T$$

$$V4.P6 = \frac{1}{2\sqrt{6}}\ EL \cong 0.204\ 124\ 145\ EL = DVF_T$$

$$V4.P7 = \frac{1}{2\sqrt{2}}\ EL \cong 0.353\ 553\ 391\ EL = DVE_T$$

$$DF(V1.V2.V3)V(V1) = \frac{1}{\sqrt{3}}\ EL \cong 0.577\ 350\ 269\ EL$$

$$DF(V1.V2.V3)V(V2) = \frac{1}{\sqrt{3}}\ EL \cong 0.577\ 350\ 269\ EL$$

$$DF(V1.V2.V3)V(V3) = \frac{1}{\sqrt{3}}\ EL \cong 0.577\ 350\ 269\ EL$$

$$DF(V1.V2.V3)E(V1.V2) = \frac{1}{2\sqrt{3}}\ EL \cong 0.288\ 675\ 135\ EL$$

$$DF(V1.V2.V4)V(V1) = \frac{\sqrt{19}}{6\sqrt{2}}\ EL \cong 0.513\ 701\ 167\ EL$$

$$DF(V1.V2.V4)V(V2) = \frac{\sqrt{19}}{6\sqrt{2}}\ EL \cong 0.513\ 701\ 167\ EL$$

$$DF(V1.V2.V4)V(V4) = \frac{1}{3\sqrt{2}}\ EL \cong 0.235\ 702\ 260\ EL$$

$$DF(V1.V2.V4)E(V1.V2) = \frac{1}{6\sqrt{2}}\ EL \cong 0.117\ 851\ 130\ EL$$

$$DF(V1.V2.V4)E(V1.V4) = \frac{\sqrt{19}}{12\sqrt{2}}\ EL \cong 0.256\ 850\ 583\ EL$$

$$DF(V1.V2.V4)E(V2.V4) = \frac{\sqrt{19}}{12\sqrt{2}}\ EL \cong 0.256\ 850\ 583\ EL$$

$$DVV(V1) = \frac{\sqrt{43}}{8\sqrt{2}}\ EL \cong 0.579\ 601\ 156\ EL$$

$$DVV(V2) = \frac{\sqrt{43}}{8\sqrt{2}}\ EL \cong 0.579\ 601\ 156\ EL$$

$$DVV(V3) = \frac{\sqrt{43}}{8\sqrt{2}}\ EL \cong 0.579\ 601\ 156\ EL$$

$$DVV(V4) = \frac{3}{8\sqrt{6}}\ EL \cong 0.153\ 093\ 109\ EL$$

$$\mathrm{DVE(V1.V2)} = \frac{\sqrt{11}}{8\sqrt{2}}\ \mathrm{EL} \cong 0.293\ 150\ 985\ \mathrm{EL}$$

$$\mathrm{DVE(V1.V3)} = \frac{\sqrt{11}}{8\sqrt{2}}\ \mathrm{EL} \cong 0.293\ 150\ 985\ \mathrm{EL}$$

$$\mathrm{DVE(V1.V4)} = \frac{\sqrt{11}}{8\sqrt{2}}\ \mathrm{EL} \cong 0.293\ 150\ 985\ \mathrm{EL}$$

$$\mathrm{DVE(V2.V3)} = \frac{\sqrt{11}}{8\sqrt{2}}\ \mathrm{EL} \cong 0.293\ 150\ 985\ \mathrm{EL}$$

$$\mathrm{DVE(V2.V4)} = \frac{\sqrt{11}}{8\sqrt{2}}\ \mathrm{EL} \cong 0.293\ 150\ 985\ \mathrm{EL}$$

$$\mathrm{DVE(V3.V4)} = \frac{\sqrt{11}}{8\sqrt{2}}\ \mathrm{EL} \cong 0.293\ 150\ 985\ \mathrm{EL}$$

$$\mathrm{DVF(V1.V2.V3)} = \frac{1}{8\sqrt{6}}\ \mathrm{EL} \cong 0.051\ 031\ 036\ \mathrm{EL}$$

$$\mathrm{DVF(V1.V2.V4)} = \frac{\sqrt{43}}{24\sqrt{2}}\ \mathrm{EL} \cong 0.193\ 200\ 385\ \mathrm{EL}$$

$$\mathrm{DVF(V1.V3.V4)} = \frac{\sqrt{43}}{24\sqrt{2}}\ \mathrm{EL} \cong 0.193\ 200\ 385\ \mathrm{EL}$$

$$\mathrm{DVF(V2.V3.V4)} = \frac{\sqrt{43}}{24\sqrt{2}}\ \mathrm{EL} \cong 0.193\ 200\ 385\ \mathrm{EL}$$

Areas:

$$V1.V2.V3 = \frac{\sqrt{3}}{4}\ EL^2 \cong 0.433\ 012\ 701\ 8\ \ EL^2$$

$$V1.V2.V4 = V2.V3.V4 = V1.V3.V4 = \frac{1}{4\sqrt{2}}\ EL^2 \cong 0.176\ 776\ 695\ EL^2$$

$$\text{Total face area} = \frac{1}{8}\left(3\sqrt{2} + 2\sqrt{3}\right)\ EL^2 \cong 0.963\ 342\ 788\ EL^2$$

Volume:

$$\text{Cubic measured volume equation} = \frac{1}{24\sqrt{2}}\ EL^3 \cong 0.029\ 462\ 783\ EL^3$$

$$\text{Synergetics' Tetravolume equation} = \frac{1}{4}\ EL^3 = 0.25\ EL^3$$

Angles:

V1.V2.V3 = V1.V3.V2 = V2.V1.V3 = 60°

$$V1.V4.V2 = V2.V4.V3 = V1.V4.V3 = 2\arccos\left(\frac{1}{\sqrt{3}}\right) \cong 109.471\ 220\ 634°$$

$$V2.V1.V4 = V1.V2.V4 = \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 35.264\ 389\ 683°$$

$$V6.V1.V4 = \arccos\left(\frac{2\sqrt{2}}{3}\right) \cong 19.471\ 220\ 634°$$

Sum of face angles = 720°.

Central Angles:

$$\text{V1.V2} = \arccos\left(\frac{-21}{43}\right) \cong 119.233\ 640\ 023°$$

$$\text{V1.V4} = \arccos\left(\frac{-1}{\sqrt{129}}\right) \cong 95.051\ 152\ 528°$$

Dihedral Angles:

$$\text{V1.V2} = \text{V2.V3} = \text{V1.V3} = \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 35.264\ 389\ 683°$$

$$\text{V1.V4} = \text{V2.V4} = \text{V3.V4} = 120°$$

Vertex Coordinates (x, y, z):

$$\text{V1} = \left(\frac{-1}{2}, \frac{-1}{2\sqrt{3}}, \frac{-1}{8\sqrt{6}}\right) \text{EL}$$

$\cong$ (– 0.5, – 0.288 675 135, – 0.051 031 036) EL

$$\text{V2} = \left(\frac{1}{2}, \frac{-1}{2\sqrt{3}}, \frac{-1}{8\sqrt{6}}\right) \text{EL}$$

$\cong$ (0.5, – 0.288 675 135, – 0.051 031 036) EL

$$\text{V3} = \left(0.0, \frac{1}{\sqrt{3}}, \frac{-1}{8\sqrt{6}}\right) \text{EL}$$

$\cong$ (0.0, 0.577 350 269, – 0.051 031 036) EL

$$\text{V4} = \left(0.0, 0.0, \frac{3}{8\sqrt{6}}\right) \text{EL}$$

$\cong$ (0.0, 0.0, 0.153 093 109) EL

Unfolded Vertex Coordinates (x, y):

$$V1 = \left(\frac{-1}{2} , 0.0 \right) EL = (-0.5 , 0.0) EL$$

$$V2 = \left(\frac{1}{2} , 0.0 \right) EL = (0.5 , 0.0) EL$$

$$V3 = \left(0.0 , \frac{\sqrt{3}}{2} \right) EL \cong (0.5 , 0.866\,025) EL$$

$$V4_1 = \left(0.0 , \frac{-1}{2\sqrt{2}} \right) EL \cong (0.5 , -0.353\,553) EL$$

$$V4_2 = \left(\frac{-\left(\sqrt{2} + \sqrt{3}\right)}{4\sqrt{2}} , \frac{1+\sqrt{6}}{4\sqrt{2}} \right) EL$$

$$\cong (-0.556\,186 , 0.609\,789) EL$$

$$V4_3 = \left(\frac{\left(\sqrt{2} + \sqrt{3}\right)}{4\sqrt{2}} , \frac{1+\sqrt{6}}{4\sqrt{2}} \right) EL$$

$$\cong (0.556\,186 , 0.609\,789) EL$$

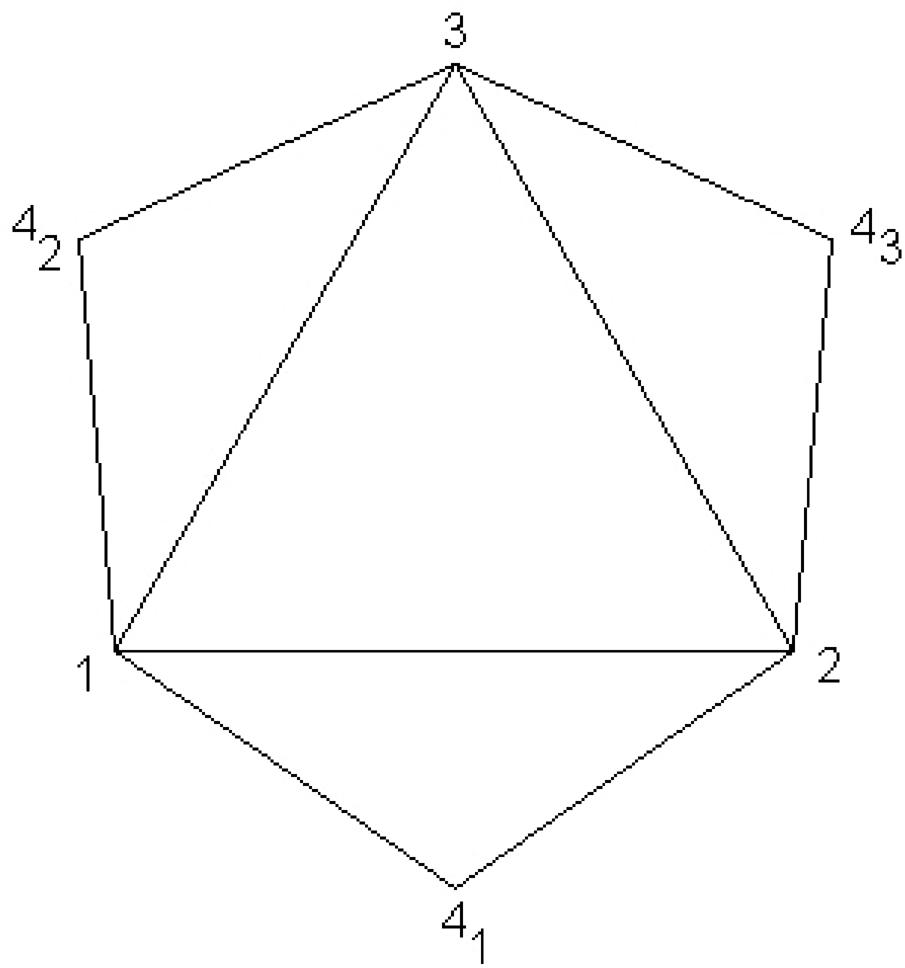

Figure 4 *Unfolded Quarter Tetrahedron.*

Comments:

The regular Tetrahedron can be divided into 4 irregular Tetrahedra, which we will call the 1/4-Tetrahedron.

The 1/4-Tetrahedron does not fill all-space.

The dual of the 1/4-Tetrahedron is another irregular Tetrahedron.