Tetrahedron

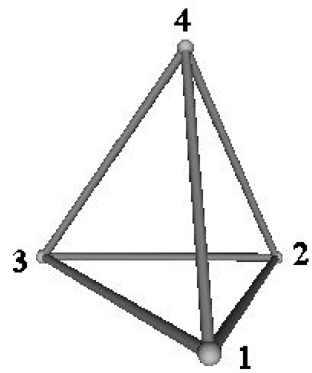

Figure 1 *Tetrahedron with vertex labels.*

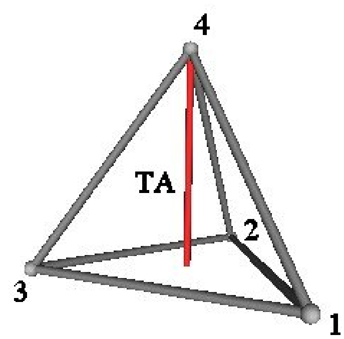

Figure 2 *Tetrahedron Altitude.*

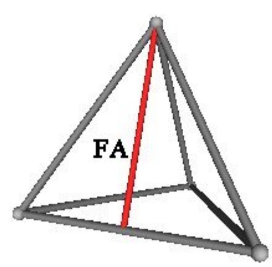

Figure 3 *Tetrahedron's Face Altitude.*

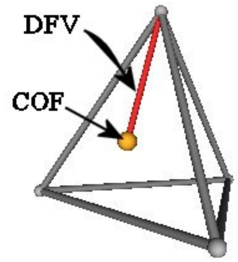

Figure 4 *Distance from a Center Of Face point to a Vertex.*

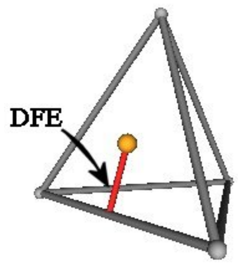

Figure 5 *Distance from a Center Of Face point to a mid-Edge point.*

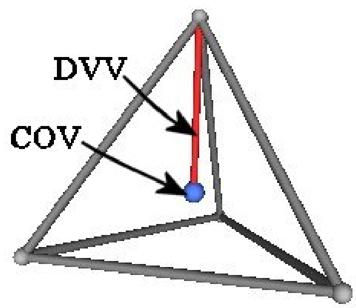

Figure 6 *Distance from the Center Of Volume to a Vertex.*

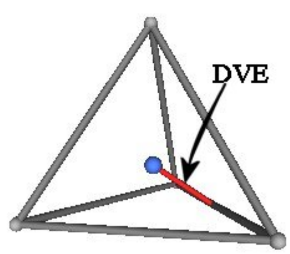

Figure 7 *Distance from the Center Of Volume to a mid-Edge point.*

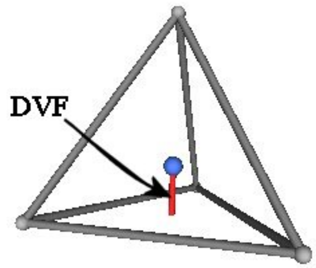

Figure 8 *Distance from the Center Of Volume to a Face center point.*

Topology:

Vertices = 4

Edges = 6

Faces = 4 equilateral triangles

Lengths:

EL ≡ Edge Length

Face Altitude = $\frac{\sqrt{3}}{2}$ EL ≅ 0.866 025 404 EL

Tetrahedron Altitude = $\frac{\sqrt{2}}{\sqrt{3}}$ EL ≅ 0.816 496 581 EL

DFV = $\frac{1}{\sqrt{3}}$ EL ≅ 0.577 350 269 EL

DFE = $\frac{1}{2\sqrt{3}}$ EL ≅0.288 675 135 EL

DVV = $\frac{\sqrt{3}}{2\sqrt{2}}$ EL ≅ 0.612 372 436 EL

DVE = $\frac{1}{2\sqrt{2}}$ EL ≅ 0.353 553 591 EL

DVF = $\frac{1}{6\sqrt{2}}$ EL ≅ 0.204 124 245 EL

Areas:

Each of the 4 equilateral triangular face have the same area.

Area of one triangular face = $\frac{\sqrt{3}}{4}$ EL^2 ≅ 0.433 012 702 EL^2

Total face area = $\sqrt{3}$ EL^2 ≅ 1.732 050 808 EL^2

Volume:

Cubic measured volume equation = $\frac{1}{2\sqrt{6}}$ EL^3 $\cong$ 0.117 851 130 EL^3

Synergetics' Tetravolume equation = EL^3

Angles:

All face angles are 60°.

Sum of face angles = 720°.

Central Angles:

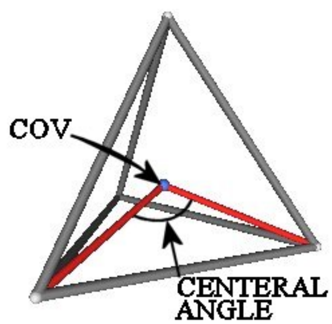

Figure 9 *Central angle.*

$$\text{All central angles are} = 2\ \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 109.471\ 220\ 634°$$

Dihedral Angles:

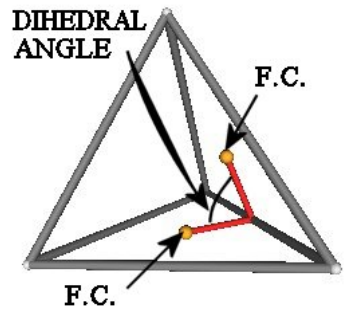

Figure 10 *Dihedral angle.*

All dihedral angles are $= \arccos\left(\frac{1}{3}\right) \cong 70.528\ 779\ 366°$

Additional Angle Information:

Note that

Central Angle + Dihedral Angle = 180°

which is the case for pairs of dual polyhedra. The Tetrahedron is self dual (is its own dual).

The angle

V4.V1.FaceCenter(V1.V2.V3) $= \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\ 610\ 317°$.

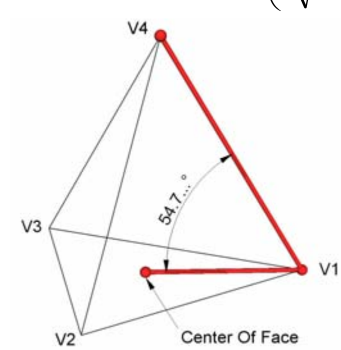

Figure 11 *Edge onto Face angle.*

When a sphere is placed around a Tetrahedron (the circumsphere) such that one of the Tetrahedron's vertices is at the "south pole", then the other 3 vertices will be on a circle which is at a latitude of

$$\theta = \arccos\left(\frac{2\sqrt{2}}{3}\right) \cong 19.471\ 220\ 634°$$

above the equator of the sphere.

When the Tetrahedron is spun about an axis through one of its vertices and through the opposite face center point, a cone is defined.

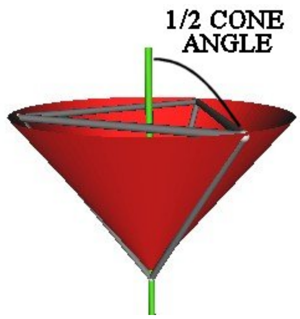

Figure 12 *Tetrahedron defined 1/2 Cone angle.*

The half-cone angle is

$$\theta = \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \cong 35.264\ 389\ 683°$$

Consider a spin axis passing through opposite mid-edge points. A cone with its apex at the center of volume and passing through the two ends of the edge used to define the edge that the spin axis passes through has a cone angle equal to the central angle.

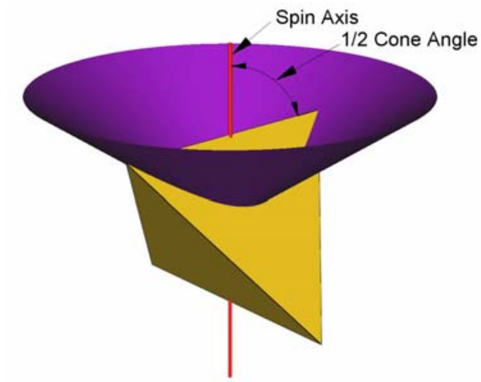

Figure 13 Another Tetrahedron defined angle.

Half of this cone angle is another Quantum Mechanic's space quantization angle for the case $j = ½$, $m_j = ½$.

$$\theta = \arccos\left(\frac{1}{\sqrt{3}}\right) \cong 54.735\,610\,317°.$$

Another cone with apex at the center of volume gives another Quantum Mechanic's space quantization angle, this time for the case $j = ½$, $m_j = – ½$.

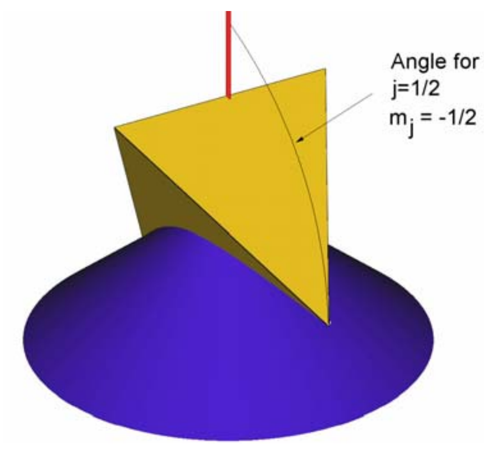

Figure 14 Another Tetrahedron defined angle.

$$\theta = \arccos\left(\frac{-1}{\sqrt{3}}\right) \cong 125.264\,389\,7°$$

Vertex Coordinates:

$$V1 = \left(\frac{-1}{2}, \frac{-1}{2\sqrt{3}}, \frac{-1}{2\sqrt{6}} \right) EL$$

$$\cong (-0.5,\ -0.288\,675\,135,\ -0.204\,124\,145)\ EL$$

$$V2 = \left(\frac{1}{2}, \frac{-1}{2\sqrt{3}}, \frac{-1}{2\sqrt{6}} \right) EL$$

$$\cong (0.5,\ -0.288\,675\,135,\ -0.204\,124\,145)\ EL$$

$$V3 = \left(0.0, \frac{1}{\sqrt{3}}, \frac{-1}{2\sqrt{6}} \right) EL$$

$$\cong (0.0,\ 0.577\,350\,269,\ -0.204\,124\,145)\ EL$$

$$V4 = \left(0.0, 0.0, \frac{3}{2\sqrt{6}} \right)$$

$$\cong (0.0,\ 0.0,\ 0.612\,372\,436)\ EL$$

Edge Map:

{{V1, V2} {V1, V3} {V1, V4} {V2, V3} {V2, V4} {V3, V4}}

Face Maps:

{{V1, V3, V2} {V1, V4, V3} {V1, V2, V4} {V2, V3, V4}}

Other Orientations:

There are 10 Tetrahedra having the same 20 vertices as 20 out of 62 vertices of the "120 Polyhedron (Type III: Dennis)". The pattern of these 20 vertex coordinate numbers is rather interesting when written in terms of the Golden Mean

$\varphi = \dfrac{1+\sqrt{5}}{2}$. In this case, the edge lengths of the Tetrahedra are

EL = $2\sqrt{2}\,\varphi^2 \cong 7.404\ 918\ 348$ units of length.

Using the vertex labeling of the 120 Polyhedron (Type III: Dennis) the vertex coordinates are as follows.

The vertex labels are those of the 120 Polyhedron.

Orientation 1:

V4 = $(0, \varphi, \varphi^3)$
$\cong (0.0, 1.618\ 033\ 989, 4.236\ 067\ 979)$
V34 = $(-\varphi, -\varphi^3, 0)$
$\cong (-1.618\ 033\ 989, -4.236\ 067\ 979, 0.0)$
V38 = $(\varphi^3, 0, -\varphi)$
$\cong (4.236\ 067\ 979, 0, -1.618\ 033\ 989)$
V47 = $(-\varphi^2, \varphi^2, -\varphi^2)$
$\cong (-2.618\ 033\ 989, 2.618\ 033\ 989, -2.618\ 033\ 989)$

Orientation 2:

V18 = $(\varphi^2, -\varphi^2, \varphi^2)$
$\cong (2.618\ 033\ 989, -2.618\ 033\ 989, 2.618\ 033\ 989)$
V23 = $(-\varphi^3, 0, -\varphi)$
$\cong (-4.236\ 067\ 979, 0.0, -1.618\ 033\ 989)$
V28 = $(-\varphi, \varphi^3, 0)$
$\cong (-1.618\ 033\ 989, 4.236\ 067\ 979, 0.0)$
V60 = $(0, -\varphi, -\varphi^3)$
$\cong (0.0, -1.618\ 033\ 989, -4.236\ 067\ 979)$

Orientation 3:

V4 $= (0, \varphi, \varphi^3)$
$\cong (0.0, 1.618\,033\,989, 4.236\,067\,979)$
V36 $= (\varphi, -\varphi^3, 0)$
$\cong (1.618\,033\,989, -4.236\,067\,979, 0.0)$
V41 $= (-\varphi^3, 0, -\varphi)$
$\cong (-4.236\,067\,979, 0, -1.618\,033\,989)$
V45 $= (\varphi^2, \varphi^2, -\varphi^2)$
$\cong (2.618\,033\,989, 2.618\,033\,989, -2.618\,033\,989)$

Orientation 4:

V16 $= (-\varphi^2, -\varphi^2, \varphi^2)$
$\cong (-2.618\,033\,989, -2.618\,033\,989, 2.618\,033\,989)$
V20 $= (-\varphi^3, 0, \varphi)$
$\cong (-4.236\,067\,979, 0.0, 1.618\,033\,989)$
V30 $= (-\varphi, \varphi^3, 0)$
$\cong (-1.618\,033\,989, 4.236\,067\,979, 0.0)$
V60 $= (0, -\varphi, -\varphi^3)$
$\cong (0.0, -1.618\,033\,989, -4.236\,067\,979)$

Orientation 5:

V8 $= (0, -\varphi, \varphi^3)$
$\cong (0.0, -1.618\,033\,989, 4.236\,067\,979)$
V28 $= (\varphi, \varphi^3, 0)$
$\cong (1.618\,033\,989, 4.236\,067\,979, 0.0)$
V41 $= (-\varphi^3, 0, -\varphi)$
$\cong (-4.236\,067\,979, 0.0, -1.618\,033\,989)$
V52 $= (\varphi^2, -\varphi^2, -\varphi^2)$
$\cong (2.618\,033\,989, -2.618\,033\,989, -2.618\,033\,989)$

Orientation 6:

V13 $= (-\varphi^2, \varphi^2, \varphi^2)$
$\cong (-2.618\ 033\ 989, 2.618\ 033\ 989, 2.618\ 033\ 989)$
V20 $= (\varphi^3, 0, \varphi)$
$\cong (4.236\ 067\ 979, 0.0, 1.618\ 033\ 989)$
V34 $= (-\varphi, -\varphi^3, 0)$
$\cong (-1.618\ 033\ 989, -4.236\ 067\ 979, 0.0)$
V56 $= (0, \varphi, -\varphi^3)$
$\cong (0.0, 1.618\ 033\ 989, -4.236\ 067\ 979)$

Orientation 7:

V8 $= (0, -\varphi, \varphi^3)$
$\cong (0.0, -1.618\ 033\ 989, 4.236\ 067\ 979)$
V30 $= (-\varphi, \varphi^3, 0)$
$\cong (-1.618\ 033\ 989, 4.236\ 067\ 979, 0.0)$
V38 $= (\varphi^3, 0, -\varphi)$
$\cong (4.236\ 067\ 979, 0.0, -1.618\ 033\ 989)$
V50 $= (-\varphi^2, -\varphi^2, -\varphi^2)$
$\cong (-2.618\ 033\ 989, -2.618\ 033\ 989, -2.618\ 033\ 989)$

Orientation 8:

V11 $= (\varphi^2, \varphi^2, \varphi^2)$
$\cong (2.618\ 033\ 989, 2.618\ 033\ 989, 2.618\ 033\ 989)$
V23 $= (-\varphi^3, 0, \varphi)$
$\cong (-4.236\ 067\ 979, 0.0, 1.618\ 033\ 989)$
V36 $= (\varphi, -\varphi^3, 0)$
$\cong (1.618\ 033\ 989, -4.236\ 067\ 979, 0.0)$
V56 $= (0, \varphi, -\varphi^3)$
$\cong (0.0, 1.618\ 033\ 989, -4.236\ 067\ 979)$

Orientation 9:

V11 $= (\ \varphi^2,\ \varphi^2,\ \varphi^2)$
$\cong$ (2.618 033 989, 2.618 033 989, 2.618 033 989)
V16 $= (-\varphi^2, -\varphi^2,\ \varphi^2)$
$\cong$ (-2.618 033 989, -2.618 033 989, 2.618 033 989)
V47 $= (-\varphi^2,\ \varphi^2, -\varphi^2)$
$\cong$ (-2.618 033 989, 2.618 033 989, -2.618 033 989)
V52 $= (\ \varphi^2, -\varphi^2, -\varphi^2)$
$\cong$ (2.618 033 989, -2.618 033 989, -2.618 033 989)

Note that by scaling by $1/\varphi^2$, Orientation 9 can be written as

V11 = (1, 1, 1)
V16 = (-1,-1, 1)
V47 = (-1, 1,-1)
V52 = (1,-1,-1)

Orientation 10:

V13 $= (-\varphi^2,\ \varphi^2,\ \varphi^2)$
$\cong$ (-2.618 033 989, 2.618 033 989, 2.618 033 989)
V18 $= (\ \varphi^2, -\varphi^2,\ \varphi^2)$
$\cong$ (2.618 033 989, -2.618 033 989, 2.618 033 989)
V45 $= (\ \varphi^2,\ \varphi^2, -\varphi^2)$
$\cong$ (2.618 033 989, 2.618 033 989, -2.618 033 989)
V50 $= (-\varphi^2, -\varphi^2, -\varphi^2)$
$\cong$ (-2.618 033 989, -2.618 033 989, -2.618 033 989)

Note that by scaling by $1/\varphi^2$, Orientation 10 can be written as

V13 = (-1, 1, 1)
V18 = (1,-1, 1)
V45 = (1, 1,-1)
V50 = (-1,-1,-1)

Unfolded Vertex Coordinates (X, Y):

V1 = (0.0, 0.0) EL

V2 = (1.0, 0.0) EL

$$V3_1 = \left(\frac{1}{2}, \frac{-\sqrt{3}}{2} \right) \text{EL} \cong (0.5,\ -0.866\,025\,4) \text{ EL}$$

$$V3_2 = \left(\frac{3}{2}, \frac{\sqrt{3}}{2} \right) \text{EL} \cong (1.5,\ 0.866\,025\,4) \text{ EL}$$

$$V3_3 = \left(\frac{-1}{2}, \frac{\sqrt{3}}{2} \right) \text{EL} \cong (-0.5,\ 0.866\,025\,4) \text{ EL}$$

$$V4 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \text{EL} \cong (0.5,\ 0.866\,025\,4) \text{ EL}$$

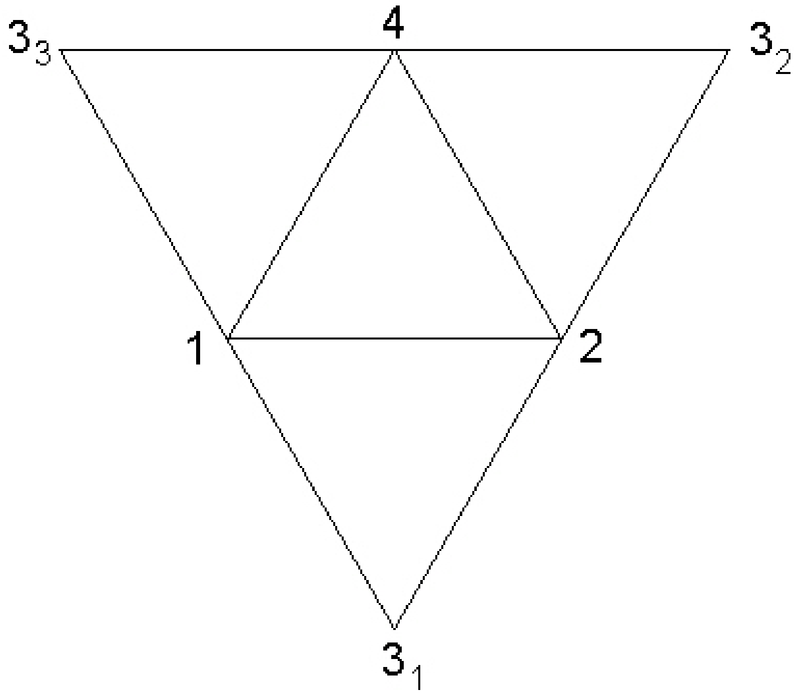

Figure 15 Layout for the Tetrahedron.

Comments:

The dual of the Tetrahedron is another Tetrahedron.

The Tetrahedron does not fill all-space by itself. It can be combined with the Octahedron to from an Octet which does fill all-space.

The Tetrahedron shares its vertices with the Cube's and the regular Dodecahedron's vertices. Ten Tetrahedra can be formed using the Dodecahedron's vertices.

The 4 face planes of the Tetrahedron are shared with 4 out of 8 face planes of the Octahedron and 4 out of 20 face planes of an Icosahedron.

Cutting the Tetrahedron with a plane that is parallel to any one of the faces results in a smaller Tetrahedron.

The Tetrahedron can be divided into 24 A Quantum Modules.

Five Tetrahedra face to face leaves a little bit of an opening. The angular amount of this opening is called the unzipping angle. The value of the angle is

$$\zeta = 360° - 5\arccos\left(\frac{1}{3}\right) \cong 7.356\ 103\ 172°$$

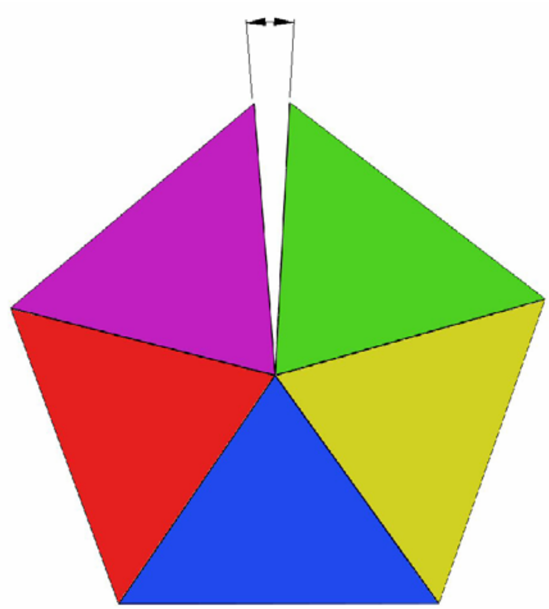

Figure 16 Unzipping angle.