

820.00 Tools of Geometry

821.01 The Early Greek geometers and their Egyptian and Babylonian predecessors pursued the science of geometry with three basic tools; the dividers, the straightedge, and the scribe. They established the first rule of the game of geometry, that they could not introduce information into their exploration unless it was acquired empirically as constructed by the use of those tools. With the progressive interactive use of these three tools, they produced modular areas, angles, and linear spaces.

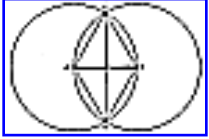
821.02 The basic flaw in their game was that they failed to identify and define as a tool the *surface* on which they inscribed. In absolute reality, this surface constituted a fourth tool absolutely essential to their demonstration. The absolute error of this oversight was missed at the time due to the minuscule size of man in relation to his planet Earth. While there were a few who conceived of Earth as a sphere, they assumed that a local planar condition existed—which the vast majority of humans assumed to be extended to infinity, with a four-cornered Earth plane surrounded by the plane of water that went to infinity.

821.03 They assumed the complementary tool to be a plane. Because the plane went to infinity in all planar directions, it could not be defined and therefore was spontaneously overlooked as a *tool* essential to their empirical demonstrating. What they could not define, yet obviously needed, they identified by the ineffable title "axiomatic," meaning "Everybody knows that." Had they recognized the essentiality of defining the fourth tool upon which they inscribed, and had they recognized that our Earth was spherical—ergo, finite; ergo, definite—they could and probably would have employed strategies completely different from that of their initiation of geometry with the exclusive use of the plane. But to the eastern Mediterranean world there lay the flat, infinite plane of the Earth at their feet on which to scratch with a scribe.

821.10 **Dividers:** The ends of two sticks can be bound together to serve as dividers. A straightedge stick could be whittled by a knife and sighted for straightness and improved by more whittling.

821.11 The opening of the dividers could be fixed by binding on a third stick between the other two ends, thus rigidifying by triangulation. Almost anyone at sea or in the desert could start playing this game.

825.00 **Greek Scribing of Right-Angle Modularity in a Plane**



[Fig. 825.01](#)

825.01 It was easy for the Greeks to use their fixed dividers to identify two points on the plane marked by the divider's two ends: A and B, respectively. Employing their straightedge, they could inscribe the line between these two points, the line AB. Using one end of the dividers as the pivot point at one end of the line, A, a circle can be described around the original line terminal: circle A. Using point B as a center, a circle can be described around it, which we will call circle B. These two circles intersect one another at two points on either side of the line AB. We will call the intersection points C and C'.

825.02 By construction, they demonstrated that points C and C' were both equidistant from points A and B. In this process, they have also defined two equilateral triangles ABC and ABC', with a congruent edge along the line AB and with points C and C' equidistant on either side from points A and B, respectively.

825.10 **Right Triangle**

825.11 They then used a straightedge to connect points C and C' with a line that they said bisected line AB perpendicularly, being generated by equidistance from either point on either side. Thus the Greeks arrived at their right triangle; in fact, their four right triangles. We will designate as point D the intersection of the lines CC' and AB. This gave the Greeks four angles around a common point. The four right triangles ADC, BDC, ADC', and BDC' have hypotenuses and legs that are, as is apparent from even the most casual inspection, of three different lengths. The leg DB, for instance, is by equidistance construction exactly one-half of AB, since AB was the radius of the two original circles whose circumferences ran through one another's centers. By divider inspections, DB is less than CD and CD is less than CB. The length of the line CD is unknown in respect to the original lines AB, BD, or AC, lines that represented the original opening of the dividers. They have established, however, with satisfaction of the rules of their game, that 360 degrees of circular unity at D could be divided into four equal 90-degree angles entirely and evenly surrounding point D.

825.20 **Hexagonal Construction**

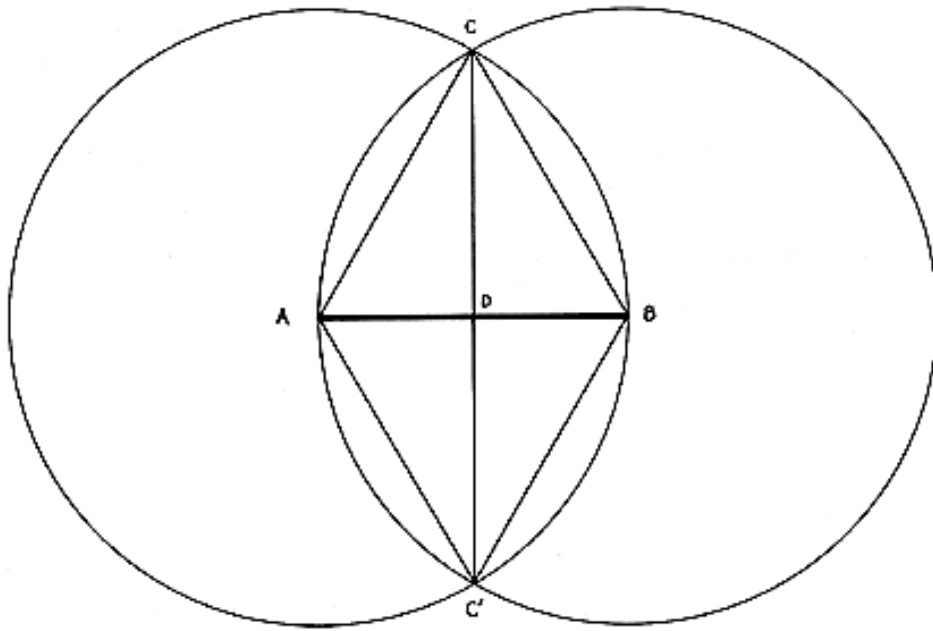
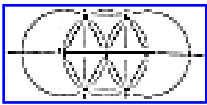


Fig. 825.01.

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825.21 **Diameter:** The Greeks then started another independent investigation with their three tools on the seemingly flat planar surface of the Earth. Using their dividers to strike a circle and using their straightedge congruent to the center of the circle, they were able with their scribe to strike a seemingly straight line through the center of construction of the circle. As the line passed out of the circle in either direction from the center, it seemingly could go on to infinity, and therefore was of no further interest to them. But inside the circle, as the line crossed the circumference at two points on either side of its center, they had the construction information that the line equated the opening of the dividers in two opposite directions. They called this line the diameter: DIA + METER.



[Fig. 825.22](#)

825.22 Now we will call the center of the constructed circle D and the two intersections of the line and the circumference A and B. That $AD = DB$ is proven by construction. They know that any point on the circumference is equidistant from D. Using their dividers again and using point A as a pivot, they drew a circle around A; they drew a second circle using B as a pivot. Both of these circles pass through D. The circle around A intersects the circle around D at two points, C and C'. The circle around B intersects the circle around D at two points, E and E'. The circle around A and the circle around B are tangent to one another at the point D.

825.23 They have now constructed four equilateral triangles in two pairs: ADC and ADC' as the first pair, and DBE and DBE' as the second pair. They know that the lines AC, CD, AC', and DC' are all identical in length, being the fixed opening of the dividers and so produced and proven by construction. The same is true of the lines DE, EB, DE', and BE'—they are all the same. The Greeks found it a tantalizing matter that the two lines CE and C'E', which lie between the vertexes of the two pairs of equilateral triangles, seemed to be equal, but there was no way for them to prove it by their construction.

825.24 At first it seemed they might be able to prove that the increments CE and C'E' are not only equal to one another, but are equal to the basic radius of the circle AD; therefore, the hexagon ACEBE'C' would be an equilateral hexagon; and hexagons would be inherently subdivisible into six 60-degree equilateral triangles around the central point, and all the angles would be of 60 degrees.

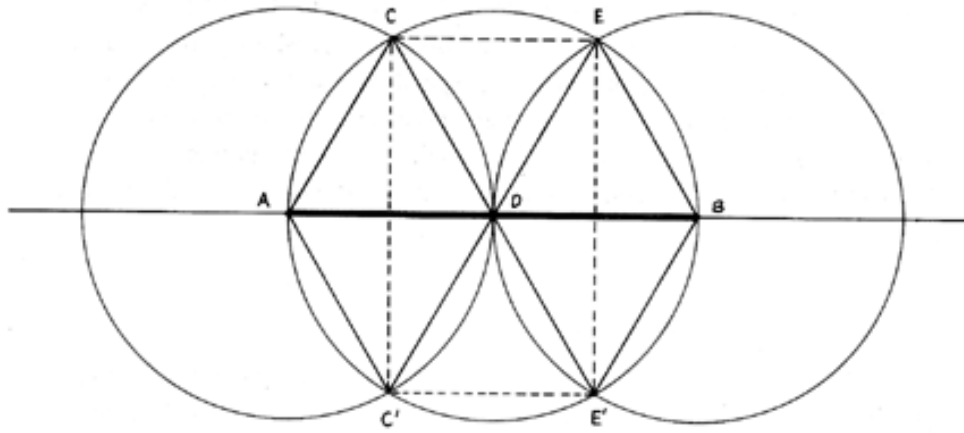


Fig. 825.22.

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825.25 There seemed to be one more chance for them to prove this to be true, which would have provided an equiangular, equiedged, triangularly stable structuring of areal mensuration. This last chance to prove it was by first showing by construction that the line ADB, which runs through the point of tangency of the circles A and B, is a straight line. This was constructed by the straightedge as the diameter of circle D. This diameter is divided by four equal half-radii, which are proven to be half-radii by their perpendicular intersection with lines both of whose two ends are equidistant from two points on either side of the intersecting lines. If it could be assumed that: (1) the lines CE and C'E' were parallel to the straight line ADB running through the point of tangency as well as perpendicular to both the lines CC' and EE'; and (2) if it could be proven that when one end between two parallels is perpendicular to one of the parallels, the other end is perpendicular to the other parallel; and (3) if it could be proven also that the perpendicular distances between any two parallels were always the same, they could then have proven $CE = CD = DE = D'E'$, and their hexagon would be equilateral and equiradial with radii and chords equal.

825.26 **Pythagorean Proof**

825.261 All of these steps were eventually taken and proven in a complex of other proofs. In the meantime, they were diverted by the Pythagoreans' construction proof of "the square of the hypotenuse of a right triangle's equatability with the sum of the squares of the other two sides," and the construction proof that any non-right triangle's dimensional values could be obtained by dropping a perpendicular upon one of its sides from one of its vertexes and thus converting it into two right triangles each of which could be solved arithmetically by the Pythagoreans' "squares" without having to labor further with empirical constructs. This arithmetical facility induced a detouring of strictly constructional explorations, hypotheses, and proofs thereof.

825.27 Due to their misassumed necessity to commence their local scientific exploration of geometry only in a supposed plane that extended forever without definable perimeter, that is, to infinity, the Ionians began using their right-triangle exploration before they were able to prove that six equilateral triangles lie in a circle around point D. They could divide the arithmetical 360 degrees of circular unity agreed upon into six 60-degree increments. And, as we have already noted, if this had been proven by their early constructions with their three tools, they might then have gone on to divide all planar space with equilateral triangles, which models would have been very convenient in connection with the economically satisfactory point-locating capability of triangulation and trigonometry.

825.28 Euclid was not trying to express forces. We, however—inspired by Avogadro's identical-energy conditions under which different elements disclosed the same number of molecules per given volume—are exploring the possible establishment of an operationally strict *vectorial geometry field*, which is an isotropic (everywhere the same) vector matrix. We abandon the Greek perpendicularity of construction and find ourselves operationally in an omnidirectional, spherically observed, multidimensional, omni- intertransforming Universe. Our first move in spherical reality scribing is to strike a quasi- sphere as the vectorial radius of construction. Our dividers are welded at a fixed angle. The second move is to establish the center. Third move: a surface circle. The radius is uniform and the lesser circle is uniform. From the triangle to the tetrahedron, the dividers go to direct opposites to make two tetrahedra with a common vertex at the center. Two tetrahedra have six internal faces=hexagon=genesis of bow tie=genesis of modelability=vector equilibrium. Only the dividers and straightedge are used. You start with two events—any distance apart: only one module with no subdivision; ergo, timeless; ergo, eternal; ergo, no frequency. Playing the game in a timeless manner. (You have to have division of the line to have frequency, ergo, to have time.) (See Secs. [420](#) and [650](#).)

825.29 Commencing proof upon a sphere as representative of energy convergent or divergent, we may construct an equilateral triangle from any point on the surface. If we describe equilateral (equiangular) triangles whose chords are identical to the radii, the same sphere may be intersected alternately by four great-circle planes whose circles intercept each other, respectively, at 12 equidistant points in such a manner that only two circles intersect at any one point. As this system is described, each great circle becomes symmetrically subdivided into six equal-arc segments whose chords are identical to the radii. From this four-dimensional tribisection, any geometrical form may be described in whole fractions.

825.30 **Two-Way Rectilinear Grid**

825.31 To the Greeks, a two-way, rectilinearly intersecting grid of parallel lines seemed simpler than would a three-way grid of parallel lines. (See Chapter 11, "Projective Transformation.") And the two-way grid was highly compatible with their practical coordinate needs for dealing with an assumedly flat-plane Universe. Thus the Greeks came to employ 90-degreeness and unique perpendicularity to the system as a basic additional dimensional requirement for the exclusive, and consequently unchallenged, three-dimensional geometrical data coordination.

825.32 Their arithmetical operations were coordinated with geometry on the assumption that first-power numbers represented linear module tallies, that second-power $N^2 =$ square increments, and that third-power $N^3 =$ cubical increments of space. First dimension was length expressed with one line. Two dimensions introduced width expressed with a cross of two lines in a plane. Three dimensions introduced height expressed by a third line crossing perpendicularly to the first two at their previous crossing, making a three-way, three-dimensional cross, which they referred to as the XYZ coordinate system. The most economical distance measuring between the peripheral points of such XYZ systems involved hypotenuses and legs of different lengths. This three-dimensionality dominated the 2,000-year scientific development of the XYZ—c.g.s. "Comprehensive Coordinate System of Scientific Mensurations." As a consequence, identifications of physical reality have been and as yet are only awkwardly characterized because of the inherent irrationality of the *peripheral* hypotenuse aspects of systems in respect to their *radial* XYZ interrelationships.

825.33 Commanded by their wealth-controlling patrons, pure scientists have had to translate their theoretical calculations of physical-system behaviors into coordinate relationship with physical reality in order to permit applied science to reduce theoretical inventions to physical practice and use. All of the analytic geometers and calculus mathematicians identify their calculus-derived coordinate behaviors of theoretical systems only in terms of linear measurements taken outwardly from central points of reference; they locate the remote event points relative to those centers only by an awkward set of perpendicularities emanating from, and parallel to, the central XYZ grid of perpendicular coordinates. The irrationality of this peripheral measuring in respect to complexedly orbited atomic nuclei has occasioned the exclusively mathematical processing of energy data without the use of conceptual models.

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