

982.80 **Closest Packing of Circles:** Because we may now give the dimensions of any sphere as $5F^n$, we have no need for π in developing spheres holistically. According to our exploratory strategy, however, we may devise one great circle of one sphere of unit rational value, and, assuming our circle also to be rational and a whole number, we may learn what the mathematical relationship to π may be—lengthwise—of our a priori circle as a whole part of a whole sphere. We know that π is the length of a circle as expressed in the diameters of the circle, a relationship that holds always to the transcendently irrational number 3.14159. But the relationship of volume 5 to the radius of one of our spheres is not altered by the circumference-to-diameter relationship because we commence with the omnidimensional wholeness of reality.

982.81 We recall also that both Newton and Leibnitz in evolving the calculus thought in terms of a circle as consisting of an infinite number of short chords. We are therefore only modifying their thinking to accommodate the manifest discontinuity of all physical phenomena as described by modern physics when we explore the concept of a circle as an aggregate of short event-vectors—tangents (instead of Newtonian short chords) whose tangential overall length must be greater than that of the circumference of the theoretical circle inscribed within those tangent event-vectors—just as Newton's chords were shorter than the circle encompassing them.

982.82 If this is logical, experimentally informed thinking, we can also consider the closest-tangential-packing of circles on a plane that produces a non-all-area-filling pattern with concave triangles occurring between the circles. Supposing we allowed the perimeters of the circles to yield bendingly outward from the circular centers and we crowded the circles together while keeping themselves as omnintegrally, symmetrically, and aggregatedly together, interpatterned on the plane with their areal centers always equidistantly apart; we would find then—as floor-tile makers learned long ago—that when closest packed with perimeters congruent, they would take on any one of three and only three possible polygonal shapes: the hexagon, the square, or the triangle—closest-packed hexagons, whose perimeters are exactly three times their diameters. Hexagons are, of course, cross sections through the vector equilibrium. The hexagon's six radial vectors exactly equal the six chordal sections of its perimeter.

982.83 Assuming the vector equilibrium hexagon to be the relaxed, cosmic, neutral, zero energy-events state, we will have the flexible but not stretchable hexagonal perimeter spun rapidly so that all of its chords are centrifugally expelled into arcs and the whole perimeter becomes a circle with its radius necessarily contracted to allow for the bending of the chords. It is this circle with its perimeter equalling six that we will now convert, first into a square of perimeter six and then into a triangle of perimeter six with the following results:

Circle	radius	0.954930	perimeter 6
Hexagon	radius	1.000000	perimeter 6 (neutral)
Square	radius	1.060660	perimeter 6
Triangle	radius	1.1546	perimeter 6

(In the case of the square, the radius is taken from the center to the corner, not the edge. In the triangle the radius is taken to the corner, not the edge.) We take particular note that the radius of the square phase of the closest-packed circle is 1.060660, the synergetics constant.

982.84 In accomplishing these transformations of the uniformly-perimetered symmetrical shapes, it is also of significance that the area of six equiangular, uniform-edged triangles is reduced to four such triangles. Therefore, it would take more equiperimeter triangular tiles or squares to pave a given large floor area than it would using equiperimetered hexagons. We thus discover that the hexagon becomes in fact the densest-packed patterning of the circles; as did the rhombic dodecahedron become the minimal limit case of self-packing allspace-filling in isometric domain form in the synergetical from-whole-to-particular strategy of discovery; while the rhombic dodecahedron is the six-dimensional state of omnidensest-packed, nuclear field domains; as did the two-frequency cube become the maximum subfrequency self-packing, allspace-filling symmetrical domain, nuclear-uniqueness, expandability and omni-intertransformable, intersymmetrical, polyhedral evolution field; as did the limit-of-nuclear-uniqueness, minimally at three-frequency complexity, self-packing, allspace-filling, semi-asymmetric octahedron of Critchlow; and maximally by the three-frequency, four-dimensional, self-packing, allspace-filling tetrakaidecahedron: these two, together with the cube and the rhombic dodecahedron constitute the only-four-is-the-limit-system set of self-packing, allspace-filling, symmetrical polyhedra. These symmetrical realizations approach a neatness of cosmic order.

983.00 **Spheres and Interstitial Spaces**

983.01 **Frequency:** In synergetics, F =

either, frequency of modular subdivision of one radius;

frequency of modular subdivision of one outer chord of a hexagonal
or, equator plane of the vector equilibrium. Thus, F = r, radius; or F = Ch,
Chord.

983.02 **Sphere Layers:** The numbers of separate spheres in each outer layer of concentric spherical layers of the vector equilibrium grows at a rate:

$$= 10r^2 + 2, \text{ or } 10F^2 + 2.$$

983.03 Whereas the space between any two concentrically parallel vector equilibria whose concentric outer planar surfaces are defined by the spheric centers of any two concentric sphere layers, is always

$$10 \left(2 + 12 \frac{r^2 - r}{2} \right),$$

or

$$10 \left(2 + 12 \frac{F^2 - F}{2} \right).$$

983.04 The difference is the nonsphere interstitial space occurring uniformly between the closest-packed spheres, which is always 6 - 5 = 1 tetrahedron.

984.00 **Rhombic Dodecahedron**

984.10 The rhombic dodecahedron is symmetrically at the heart of the vector equilibrium. The vector equilibrium is the ever-regenerative, palpitable heart of all the omniresonant physical-energy hearts of Universe.

985.00 **Synergetics Rational Constant Formulas for Area of a Circle and Area and Volume of a Sphere**

985.01 We employ the synergetics constant "S," for correcting the cubical XYZ coordinate inputs to the tetrahedral inputs of synergetics:

$$S^1 = 1.060660$$

$$S^2 = 1.12487$$

$$S^3 = 1.1931750$$

We learn that the sphere of radius 1 has a "cubical" volume of 4.188; corrected for tetrahedral value we have $4.188 \times 1.193 = 4.996 = 5$ tetrahedra = 1 sphere.

Applying the S^2 to the area of a circle of radius 1, ($\pi = 3.14159$) $3.14159 \times 1.125 = 3.534$ for the corrected "square" area.

985.02 We may also employ the XYZ to synergetics conversion factors between commonly based squares and equiangular triangles: from a square to a triangle the factor is 2.3094; from a triangle to a square the factor is 0.433. The constant π $3.14159 \times 2.3094 = 7.254 = 7 \frac{1}{4}$; thus $7 \frac{1}{4}$ triangles equal the area of a circle of radius 1. Since the circle of a sphere equals exactly four circular areas of the same radius, $7 \frac{1}{4} \times 4 = 29 =$ area of the surface of a sphere of radius 1.

985.03 The area of a hexagon of radius 1 shows the hexagon with its vertexes lying equidistantly from one another in the circle of radius 1 and since the radii and chords of a hexagon are equal, then the six equilateral triangles in the hexagon plus $1 \frac{1}{4}$ such triangles in the arc-chord zones equal the area of the circle: $1.25/6 = 0.208$ zone arc-chord area. Wherefore the area of a circle of frequency 2 = 29 triangles and the surface of a sphere of radius 2 = 116 equilateral triangles.

985.04 For the 120 LCD spherical triangles $S = 4$; $S = 4$ for four great-circle areas of the surface of a sphere; therefore S for one great-circle area equals exactly one spherical triangle, since $120/4 = 30$ spherical triangles vs. $116/4 = 29$ equilateral triangles. The S disparity of 1 is between a right spherical triangle and a planar equiangular triangle. Each of the 120 spherical LCD triangles has exactly six degrees of spherical excess, their three corners being 90 degrees, 60 degrees and 36 degrees vs. 90 degrees, 60 degrees, 30 degrees of their corresponding planar triangle. Therefore, 6 degrees per each spherical triangle times 120 spherical triangles amounts to a total of 720 degrees spherical excess, which equals exactly one tetrahedron, which exact excessiveness elucidates and elegantly agrees with previous discoveries (see Secs. [224.07](#), [224.10](#), and [224.20](#)).

985.05 The synergetical definition of an operational sphere (vs. that of the Greeks) finds the spheric experience to be operationally always a star-point-vertexed polyhedron, and there is always a 720 degree (one tetrahedron) excess of the Greek's sphere's assumption of 360 degrees around each vertex vs. the operational sum of the external angles of any system, whether it be the very highest frequency (seemingly "pure" spherical) regular polyhedral system experience of the high-frequency geodesic spheres, or irregular giraffe's or crocodile's chordally-interconnected, outermost-skin-points-defined, polyhedral, surface facets' corner-angle summation.

985.06 Thus it becomes clear that $S = 1$ is the difference between the infinite frequency series' perfect nuclear sphere of volume S and 120 quanta modules, and the four-whole-great-circle surface area of 116 equilateral triangles, which has an exact spherical excess of 720 degrees = one tetrahedron, the difference between the 120 spherical triangles and the 120 equilateral triangles of the 120-equiplanar-faceted polyhedron.

985.07 This is one more case of the *one tetrahedron: one quantum jump* involved between various stages of nuclear domain intertransformations, all the way from the difference between integral-finite, nonsimultaneous, scenario Universe, which is inherently nonunitarily conceptual, and the maximum-minimum, conceptually thinkable, systemic subdivision of Universe into an omnirelevantly frequenced, tunable set which is always one positive tetrahedron (macro) and one negative tetrahedron (micro) less than Universe: the definitive conceptual vs. finite nonunitarily conceptual Universe (see Secs. [501.10](#) and [620.01](#)).

985.08 The difference of *one* between the spheric domain of the rhombic dodecahedron's *six* and the nuclear sphere's *five*—or between the tetra volume of the octahedron and the three-tetra sections of the tetrahelix—these are the *prime wave pulsation propagating quanta phenomena* that account for local aberrations, twinkle angles, and unzipping angles manifest elsewhere and frequently in this book.

985.10 **Table: Triangular Area of a Circle of Radius 1** $F^1 = \text{Zero-one}$
 frequency = $7 \frac{1}{4}$.

Table of whole triangles only with $F = \text{Even } N$, which is because $\text{Even } N = \text{closed}$
 wave circuit.

	F^N	F^2		Triangular Areas of Circle of Radius 1:	
Open	1	1	$\times 7 \frac{1}{4}$.	$7 \frac{1}{4}$.	
Close	2	4	$\times 7 \frac{1}{4}$.	29	
Open	3	9	$\times 7 \frac{1}{4}$. ($63 + 2 \frac{1}{4}$.)	$65 \frac{1}{4}$.	
Close	4	16	$\times 7 \frac{1}{4}$. ($112 + 4$)	116	(also surface of one sphere)
Open	5	25	$\times 7 \frac{1}{4}$. ($175 + 6 \frac{1}{4}$.)	$181 \frac{1}{4}$.	
Close	6	36	$\times 7 \frac{1}{4}$. ($252 + 9$)	261	

985.20 **Spheric Experience:** Experientially defined, the spheric experience, i.e., a sphere, is an aggregate of critical-proximity event "points." Points are a multidimensional set of crossings of orbits: tracteries, foci, fixes, vertexes coming cometlike almost within intertouchability and vertexing within cosmically remote regions. Each point consists of three or more vectorially convergent events approximately equidistant from one approximately locatable and as yet nondifferentially resolved, point; i.e., three or more visualizable, four-dimensional vectors' most critical proximity, convergently-divergently interpassing region, local, locus, terminal and macrocosmically the most complex of such point events are the celestial stars; i.e., the highest-speed, high-frequency energy event, importing-exporting exchange centers. Microcosmically the atoms are the inbound terminals of such omniorderly exchange systems.

985.21 Spheres are further cognizable as vertexial, star-point-defined, polyhedral, constellar systems structurally and locally subdividing Universe into insiderness and outsiderness, microcosm-macrocosm.

985.22 Physically, spheres are high-frequency event arrays whose spheric complexity and polyhedral system unity consist structurally of discontinuously islanded, critical-proximity-event huddles, compressionally convergent events, only tensionally and omni-interattractively cohered. The pattern integrities of all spheres are the high- frequency, traffic-described subdivisionings of either tetrahedral, octahedral, or icosahedral angular interference, intertriangulating structures profiling one, many, or all of their respective great-circle orbiting and spinning event characteristics. All spheres are highfrequency geodesic spheres, i.e., triangular-faceted polyhedra, most frequently icosahedral because the icosasphere is the structurally most economical.

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