

## 986.060 **Characteristics of Tetrahedra**



[Fig. 986.061](#)

986.061 The tetrahedron is at once both the simplest system and the simplest structural system in Universe (see Secs. [402](#) and [620](#)). All systems have a minimum set of topological characteristics of vertexes, faces, and edges (see Secs. [1007.22](#) and [1041.10](#)). Alteration of the minimum structural system, the tetrahedron, or any of its structural- system companions in the primitive hierarchy (Sec. [982.61](#)), may be accomplished by either external or internal contact with other systems—which other systems may cleave, smash, break, or erode the simplest primitive systems. Other such polyhedral systems may be transformingly developed by wind-driven sandstorms or wave-driven pebble beach actions. Those other contacting systems can alter the simplest primitive systems in only two topological-system ways:

1. by truncating a vertex or a plurality of vertexes, and
2. by truncating an edge or a plurality of edges.

Faces cannot be truncated. (See Fig. [986.061](#).)



[Fig. 1086.062](#)

986.062 As we have learned regarding the "Platonic solids" carvable from cheese (Sec. [623.10](#)), slicing a polyhedron parallel to one of its faces only replaces the original face with a new face parallel to the replaced face. Whereas truncating a vertex or an edge eliminates those vertexes and edges and replaces them with faces—which become additional faces effecting a different topological abundance inventory of the numbers of vertexes and edges as well. For every edge eliminated by truncation we gain two new edges and one new face. For every corner vertex eliminated by truncation our truncated polyhedron gains three new vertexes, three new edges, and one new face.

986.063 The cheese tetrahedron (Sec. [623.13](#)) is the only one of the primitive hierarchy of symmetrical polyhedral systems that, when sliced parallel to only one of its four faces, maintains its symmetrical integrity. It also maintains both its primitive topological and structural component inventories when asymmetrically sliced off parallel to only one of its four disparately oriented faces. When the tetrahedron has one of its vertexes truncated or one of its edges truncated, however, then it loses its overall system symmetry as well as both its topological and structural identification as the structurally and topologically simplest of cosmic systems.

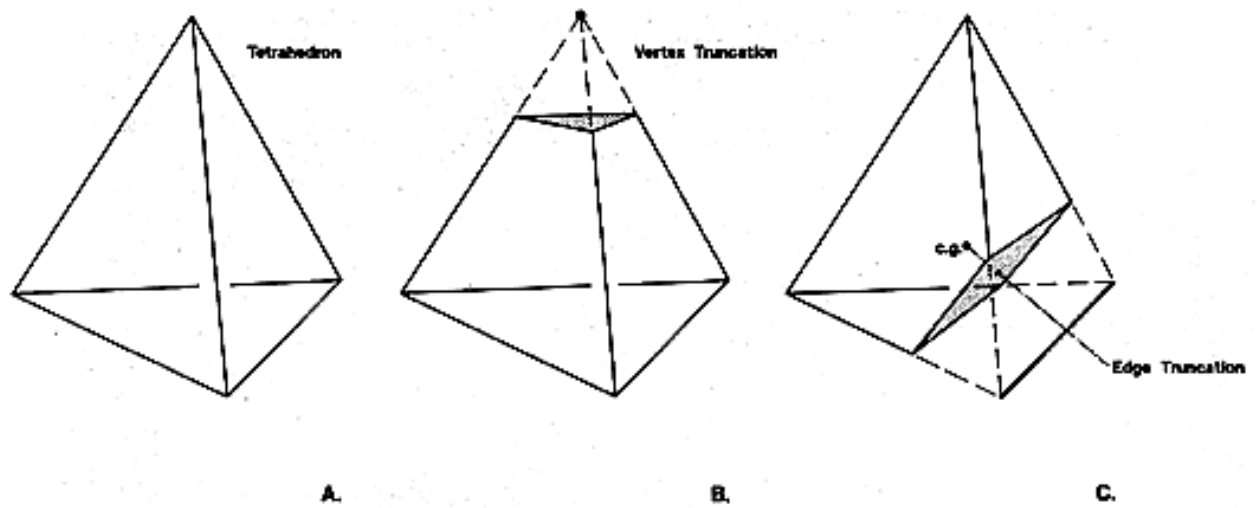


Fig. 986.061 Truncation of Tetrahedra: Only vertexes and edges may be truncated. (Compare Figs. [987.241](#) and [1041.11.](#))

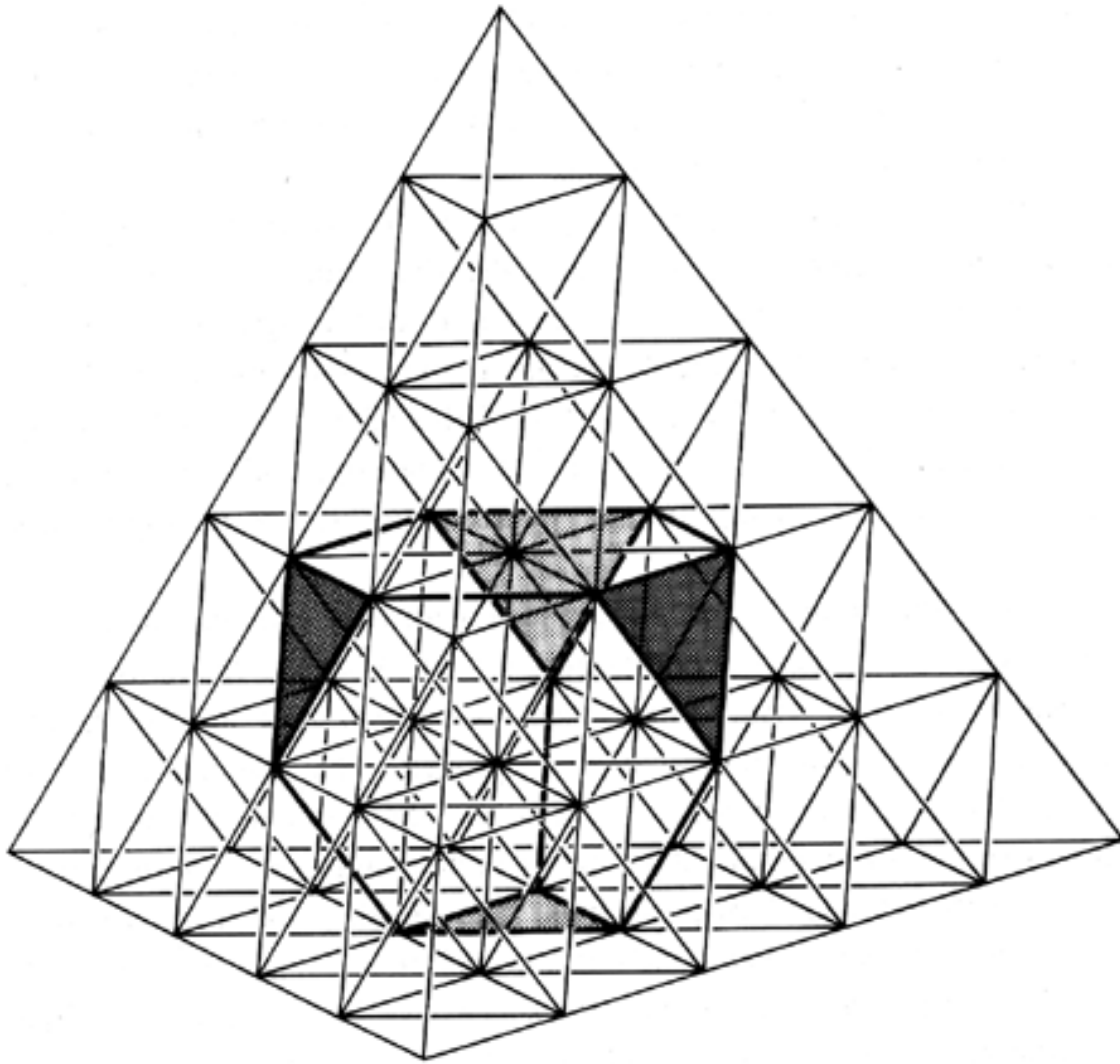


Fig. 986.062 Truncated Tetrahedron within Five-frequency Tetra Grid: Truncating the vertexes of the tetrahedron results in a polyhedron with four triangular faces and four hexagonal faces. (Compare Figs. [1041.11](#) and [1074.13](#).)

986.064 We may now make a generalized statement that the simplest system in Universe, the tetrahedron, can be design-altered and lose its symmetry only by truncation of one or more of its corners or edges. If all the tetrahedron's four vertexes and six edges were to be similarly truncated (as in Fig. [1041.11](#)) there would result a symmetrical polyhedron consisting of the original four faces with an addition of 10 more, producing a 14-faceted symmetrical polyhedron known as the tetrakaidecahedron, or Kelvin's "solid," which (as shown in Sec. [950.12](#) and Table [954.10](#)) is an allspace filler—as are also the cube, the rhombic dodecahedron, and the tetrahedral Mites, Sytes, and Couplers. All that further external alteration can do is produce more vertex and edge truncations which make the individual system consist of a greater number of smaller-dimension topological aspects of the system. With enough truncations—or knocking off of corners or edges—the system tends to become less angular and smoother (smoother in that its facets are multiplying in number and becoming progressively smaller and thus approaching subvisible identification). Further erosion can only "polish off" more of the only-microscopically- visible edges and vertexes. A polished beach pebble, like a shiny glass marble or like a high-frequency geodesic polyhedral "spheric" structure, is just an enormously high- frequency topological inventory-event system.

986.065 **Joints, Windows, and Struts:** As we have partially noted elsewhere (Secs. [536](#) and [604](#)), Euler's three primitive topological characteristics—*texes*, *faces*, and *lines*—are structurally identifiable as *joints*, *windows*, and push-pull *struts*, respectively. When you cannot see through the windows (faces), it is because the window consists of vast numbers of subvisible windows, each subvisible-magnitude window being strut- mullion-framed by a complex of substructural systems, each with its own primitive topological and structural components.

986.066 Further clarifying those structural and topological primitive componentation characteristics, we identify the structural congruences of two or more joined-together- systems' components as two congruent single vertexes (or joints) producing one single, univalent, universal-joint intersystem bonding. (See Secs. [704](#), [931.20](#), and Fig. [640.41B](#).) Between two congruent pairs of interconnected vertexes (or joints) there apparently runs only one apparent (because congruent) line, or interrelationship, or push-pull strut, or hinge.

986.067 Returning to our early-Greek geometry initiative and to the as-yet-persistent academic misconditioning by the Greeks' oversights and misinterpretations of their visual experiences, we recall how another non-Ionian Greek, Pythagoras, demonstrated and "proved" that the number of square areas of the unit-module-edged squares and the number of cubical module volumes of the unit-module-edged cubes correspond exactly with arithmetic's second-powerings and third-powerings. The Greeks, and all mathematicians and all scientists, have ever since misassumed these square and cube results to be the only possible products of such successive intermultiplying of geometry's unit-edge-length modular components. One of my early mathematical discoveries was the fact that all triangles—regular, isosceles, or scalene—may be modularly subdivided to express second-powering. Any triangle whose three edges are each evenly divided into the same number of intervals, and whose edge-interval marks are cross-connected with lines that are inherently parallel to the triangle's respective three outer edges—any triangle so treated will be subdivided by little triangles all exactly similar to the big triangle thus subdivided, and the number of small similar triangles subdividing the large master triangle will always be the second power of the number of edge modules of the big triangle. In other words, we can say "triangling" instead of "squaring," and since all squares are subdivisible into two triangles, and since each of those triangles can demonstrate areal second-powering, and since nature is always most economical, and since nature requires structural integrity of her forms of reference, she must be using "triangling" instead of "squaring" when any integer is multiplied by itself. (See Sec. [990](#).)

986.068 This seemed to be doubly confirmed when I discovered that any nonequiedged quadrangle, with each of its four edges uniformly subdivided into the same number of intervals and with those interval marks interconnected, produced a pattern of dissimilar quadrangles. (See Fig. [990.01](#).) In the same manner I soon discovered experimentally that all tetrahedra, octahedra, cubes, and rhombic dodecahedra—regular or skew—could be unitarily subdivided into tetrahedra with the cube consisting of three tetra, the octahedron of four tetra, and the rhombic dodecahedron of six similar tetra; and that when any of these regular or skew polyhedras' similar or dissimilar edges and faces were uniformly subdivided and interconnected, their volumes would always be uniformly subdivided into regular or skew tetrahedra, and that  $N^3$  could and should be written and spoken of as  $N^{\text{tetrahedroned}}$  and not as  $N^{\text{cubed}}$ .

986.069 Nature would use the tetrahedron as the module of subdivision because nature has proven to the physicists and the other physical scientists that she always chooses the most economic realization. Cubes require three times as much Universe as do tetrahedra to demonstrate volumetric content of systems because cubic identification with third-powering used up three times as much volume as is available in Universe. As a result of cubic mensuration science has had to invent such devices as "probability" and "imaginary numbers." Thus "squaring" and "cubing," instead of nature's "triangling" and "tetrahedroning," account for science's using mathematical tools that have no physical- model demonstrability—ergo, are inherently "unscientific."

#### 986.070 **Buildings on Earth's Surface**

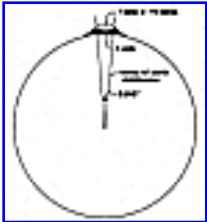
986.071 In the practical fortress and temple building of the earliest known Mesopotamians, Egyptians, and Greeks their cubes and omnirectilinear blocks seemed readily to fill allspace as they were assembled into fortress or temple walls with plumb bobs, water-and-bubble levels, straightedges, and right-triangle tools. No other form they knew—other than the cube—seemed to fill allspace as demonstrated in practical masonry; wherefore they assumed this to be scientifically demonstrated proof of the generalizability of their mathematically abstracted plane- and solid-geometry system and its XYZ coordination.

986.072 Because of the relatively diminutive size of humans in respect to the size of our planet, world-around society as yet spontaneously cerebrates only in terms of our immediate world's seeming to demonstrate itself to be a flat plane base, all of the perpendiculars of which—such as trees and humans and human-built local structures- appear to be rising from the Earth parallel to one another—ergo, their ends point in only two possible directions, "up" or "down." . . . It's "a wide, wide world," and "the four corners of the Earth."

986.073 It was easy and probably unavoidable for humanity to make the self-deceptive blunders of assuming that a cube held its shape naturally, and not because the stone-cutters or wood-cutters had chosen quite arbitrarily to make it in this relatively simple form. Human's thought readily accepted—and as yet does—the contradictory abstract state "solid." The human eye gave no hint of the energetic structuring of the atomic microcosm nor of the omnidynamic, celestial-interpositioning transformations of both macro- and micro-Universe.

986.074 Prior to steel-framed or steel-reinforced-concrete construction methods, humans' buildings that were constructed only of masonry could not be safely built to a height of over 20 stories—approximately 200 feet high. Such a masonry building was Chicago's turn-of-the-20th-century world-record Monadnock Building, whose base covered a small but whole city block. It is not until we reach a height of 100 stories—approximately 1000 feet high—that two exactly vertical square columns, each with base edges of 250 feet, built with exactly vertical walls, and touching one another only along one of each of their base edges, will show a one-inch space between them. The rate their vertical walls part from one another is only 1/1000th of an inch for each foot of height.

986.075 Masons' and carpenters' linear measuring devices are usually graduated only to 1/16th of an inch, and never finer than 1/32nd of an inch. Thus differentials of a thousandth of an inch are undetectable and are altogether inadvertently overlooked; ergo, they get inadvertently filled-in, or cross-joined, never to have been known to exist even on the part of the most skilled and conscientious of building craftsmen, whose human eyes cannot see intervals of less than 1/100th of an inch.



[Fig. 986.076](#)

986.076 If two exactly-vertical-walled city skyscrapers are built side by side, not until they are two and one-half miles high (the height of Mount Fuji) will there be a space of one foot between the tops of their two adjacent walls. (See Fig. [986.076](#).) Of course, the farther apart the centers of their adjacent bases, the more rapidly will the tops of such high towers veer away from one another:

The twin towers of New York's Verrazano Bridge are 693 feet high . . . soaring as high as a 70-story skyscraper . . . set almost a mile from each other, the two towers, though seemingly parallel, are an inch and five-eighths farther apart at their summits than at their bases because of the Earth's curvature.<sup>2</sup>

(Footnote 2: *The Engineer* (New York: Time-Life Books, 1967.) If the towers are 12,000 miles apart—that is, halfway around the world from one another—their tops will be built in exactly opposite directions ergo, at a rate of two feet farther apart for each foot of their respective heights.)

986.077 It is easy to understand how humans happened to think it "illogical" to have to consider that all the perpendiculars to a sphere are radii of that sphere—ergo, never parallel to one another. Our humans-in-Universe scale is inherently self-deceptive—ergo, difficult to cope with rigorously.

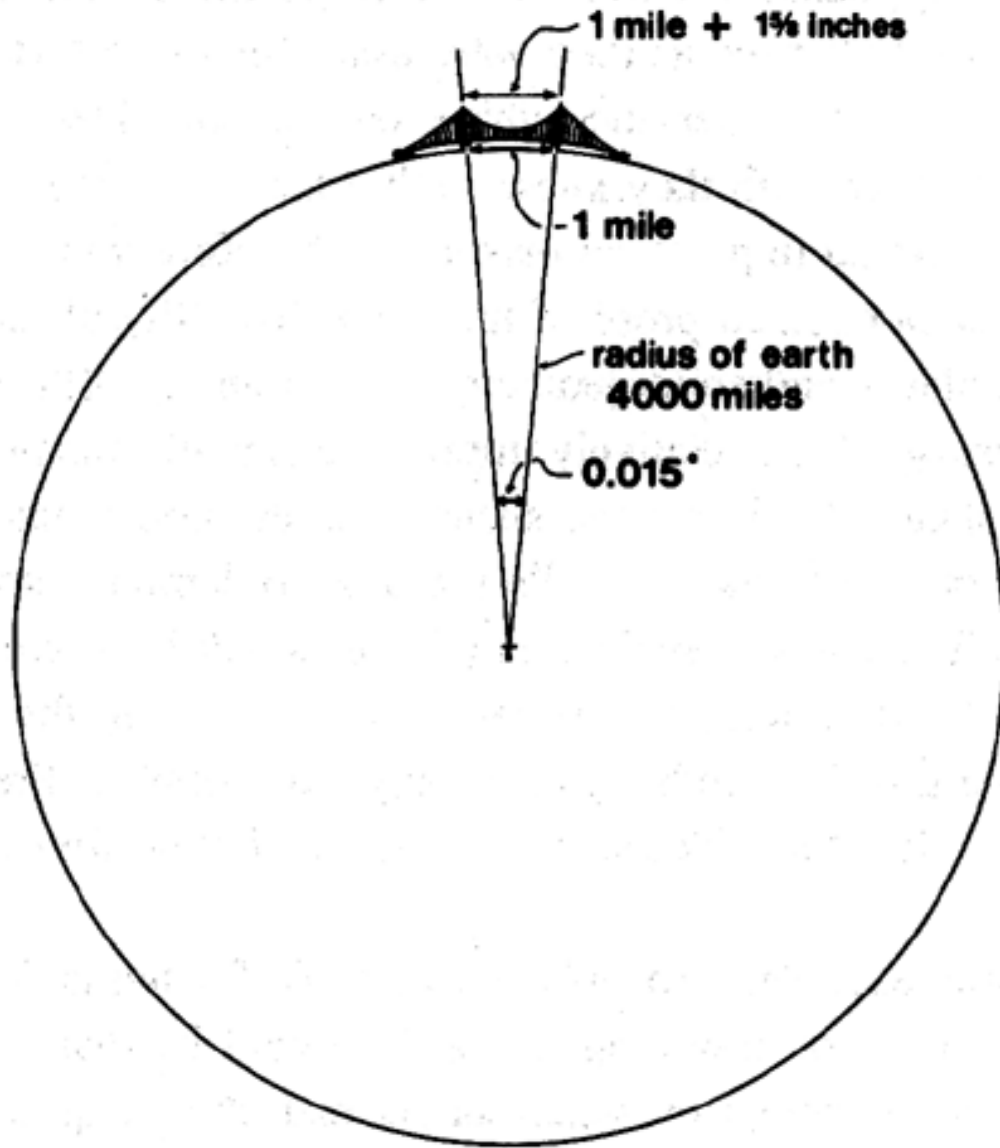


Fig. 986.076 Diagram of Verrazano Bridge: The two towers are not parallel to each other.



## 986.080 **Naive Perception of Childhood**

986.081 The inventory of experimentally demonstrated discoveries of science which had accrued by the time of my childhood gave me reason to question many of the "abstractions" of geometry as I was being instructed in that subject. Axioms were based on what only seemed "self-evident," such as the stone block or the "cubical" wooden play blocks of my nursery. To society they "obviously held their shape." I do not think that I was precocious or in any way a unique genius. I had one brother; he was three years younger than I. His eyesight was excellent; mine was atrocious. I did not get my first eyeglasses until my younger brother was running around and talking volubly. He could see things clearly; I could not. Our older sister could also see things clearly. I literally had to feel my way along—tactilely—in order to recognize the "things" of my encountered environment—ergo, my deductions were slow in materializing. My father called my younger brother "stickly-prickly" and he called me "slow-and-solid"—terms he adopted from "The Jaguar and the Armadillo" in Kipling's *Just So Stories*.

986.082 I was born cross-eyed on 12 July 1895. Not until I was four-and-a-half years old was it discovered that I was also abnormally farsighted. My vision was thereafter fully corrected with lenses. Until four-and-a-half I could see only large patterns—houses, trees, outlines of people—with blurred coloring. While I saw two dark areas on human faces, I did not see a human eye or a teardrop or a human hair until I was four. Despite my newly gained ability—in 1899—to apprehend details with glasses, my childhood's spontaneous dependence upon only big-pattern clues has persisted. All that I have to do today to reexperience what I saw when I was a child is to take off my glasses, which, with some added magnification for age, have exactly the same lens corrections as those of my first five-year-old pair of spectacles. This helps me to recall vividly my earliest sensations, impressions, and tactical assumptions.

986.083 I was sent to kindergarten before I received my first eyeglasses. The teacher, Miss Parker, had a large supply of wooden toothpicks and semidried peas into which you could easily stick the sharp ends of the toothpicks. The peas served as joints between the toothpicks. She told our kindergarten class to make structures. Because all of the other children had good eyesight, their vision and imagination had been interconditioned to make the children think immediately of copying the rectilinearly framed structures of the houses they saw built or building along the road. To the other children, horizontally or perpendicularly parallel rectilinear forms were structure. So they used their toothpicks and peas to make cubic and other rectilinear models. The semidried peas were strong enough to hold the angles between the stuck-in toothpicks and therefore to make the rectilinear forms hold their shapes—despite the fact that a rectangle has no inherent self-structuring capability.

986.084 In my poor-sighted, feeling-my-way-along manner I found that the triangle—I did not know its name—was the only polygon—I did not know that word either—that would hold its shape strongly and rigidly. So I naturally made structural systems having interiors and exteriors that consisted entirely of triangles. Feeling my way along I made a continuous assembly of octahedra and tetrahedra, a structured complex to which I was much later to give the contracted name "octet truss." (See Sec. [410.06](#)). The teacher was startled and called the other teachers to look at my strange contriving. I did not see Miss Parker again after leaving kindergarten, but three-quarters of a century later, just before she died, she sent word to me by one of her granddaughters that she as yet remembered this event quite vividly.

986.085 Three-quarters of a century later, in 1977, the National Aeronautics and Space Administration (NASA), which eight years earlier had put the first humans on the Moon and returned them safely to our planet Earth, put out bids for a major space-island platform, a controlled-environment structure. NASA's structural specifications called for an "octet truss" —my invented and patented structural name had become common language, although sometimes engineers refer to it as "space framing." NASA's scientific search for the structure that had to provide the most structural advantages with the least pounds of material—ergo, least energy and seconds of invested time—in order to be compatible and light enough to be economically rocket-lifted and self-erected in space—had resolved itself into selection of my 1899 octet truss. (See Sec. [422](#).)

986.086 It was probable also that my only-insectlike, always-slow, cross-referencing strategy of touching, tasting, smelling, listening, and structurally testing by twisting and pounding and so forth—to which I spontaneously resorted—made me think a great deal about the fact that- when I broke a piece of glass or a stone or a wooden cube apart, it did not separate naturally into little cubes but usually into sharp pointed shapes. In the earliest of my memories I was always suspicious of the integrity of cubes, which only humans seemed to be introducing into the world. There were no cubical roses, eggs, trees, clouds, fruits, nuts, stones, or anything else. Cubes to me were unnatural: I observed humans deliberately sawing ice into large rectilinear cakes, but window glass always broke itself into predominantly triangular pieces; and snowflakes formed themselves naturally into a myriad of differently detailed, six-triangled, hexagonal patterns.

986.087 I was reacting normally in combining those spontaneous feelings of my childhood with the newly discovered knowledge of the time: that light has speed (it is not instantaneous, and comes in smallest packages called photons); that there is something invisible called electricity (consisting of "invisible behaviors" called electrons, which do real work); and that communication can be wireless, which Marconi had discovered the year I was born—and it is evident that I was reacting normally and was logically unable to accept the customarily honored axioms that were no longer "self-evident."

986.088 My contemporaries and I were taught that in order to design a complete and exact sphere and have no materials left over, we must employ the constant known as pi ( $\pi$ ), which I was also taught was a "transcendentally irrational number," meaning it could *never be resolved*. I was also informed that a singly existent bubble was a sphere; and I asked, To how many places does nature carry out pi when she makes each successive bubble in the white-crested surf of each successive wave before nature finds out that pi can never be resolved? . . . And at what moment in the making of each separate bubble in Universe does nature decide to terminate her eternally frustrated calculating and instead turn out a fake sphere? I answered myself that I don't think nature is using pi or any of the other irrational fraction constants of physics. Chemistry demonstrates that nature always associates or disassociates in whole rational increments.... Those broken window shards not only tended to be triangular in shape, but also tended to sprinkle some very fine polyhedral pieces. There were wide ranges of sizes of pieces, but there were no pieces that could not "make up their minds" or resolve which share of the original whole was theirs. Quite the contrary, they exploded simultaneously and

unequivocally apart.

986.089 At first vaguely, then ever more excitedly, precisely, and inclusively, I began to think and dream about the optimum grand strategy to be employed in discovering nature's own obviously elegant and exquisitely exact mathematical coordinate system for conducting the energetic transactions of eternally regenerative Universe. How does nature formulate and mass-produce all the botanical and zoological phenomena and all the crystals with such elegant ease and expedition?

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