

986.480 **Consideration 13: Correspondence of Surface Angles and Central Angles**

986.481 It was next to be noted that spherical trigonometry shows that nature's smallest common denominator of system-surface subdivision by any one type of the seven great-circle-symmetry systems is optimally accomplished by the previously described 120 spherical-surface triangles formed by the 15 great circles, whose central angles are approximately

20.9°

37.4°

31.7°

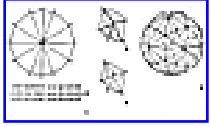
90°

whereas their surface angles are 36 degrees at A, 60 degrees at B, and 90 degrees at C.

986.482 We recall that the further self-subdividing of the 120 triangles, as already defined by the 15 great circles and as subdividingly accomplished by the icosahedron's additional 6- and 10-great-circle spinnabilities, partitions the 120 LCD triangles into 480 right triangles of four types: ADC, CDE, CFE, and EFB—with 60 positive and 60 negative pairs of each. (See Figs. [901.03](#) and [986.314](#).) We also recall that the 6- and 10-great-circle-spun hemispherical gridding further subdivided the 120 right triangles—ACB—formed by the 15 great circles, which produced a total of 12 types of surface angles, four of them of 90 degrees, and three whose most acute angles subdivided the 90-degree angle at C into three surface angles: ACD—31.7 degrees; DCE—37.4 degrees; and ECB—20.9 degrees, which three *surface angles*, we remember, correspond exactly to the three *central angles* COB, BOA, and COA, respectively, of the triacontahedron's tetrahedral T Quanta Module ABCO^t.

986.500 **E Quanta Module**

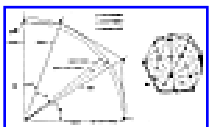
986.501 **Consideration 14: Great-circle Foldable Discs**



[Fig. 986.502](#)

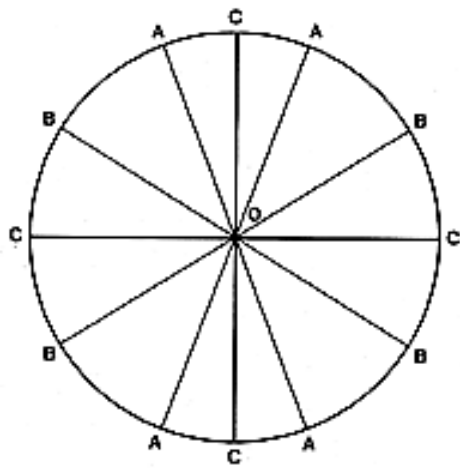
986.502 With all the foregoing events, data, and speculative hypotheses in mind, I said I think it would be worthwhile to take 30 cardboard great circles, to divide them into four 90-degree quadrants, then to divide each of the quadrants into three angles—COA, 20.9 degrees; AOB, 37.4 degrees; and BOC, 31.7 degrees—and then to score the cardboard discs with fold lines in such a manner that the four lines CO will be negatively outfolded, while the lines AO and BO will be positively infolded, so that when they are altogether folded they will form four similar-arc-edged tetrahedra ABCO with all of their four CO radii edges centrally congruent. And when 30 of these folded great-circle sets of four T Quanta Module tetrahedra are each triple-bonded together, they will altogether constitute a sphere. This spherical assemblage involves pairings of the three intercongruent interface triangles AOC, COB, and BOA; that is, each folded great-circle set of four tetra has each of its four internal triangular faces congruent with their adjacent neighbor's corresponding AOC, COB, and BOC interior triangular faces. (See Fig. [986.502](#).)

986.503 I proceeded to make 30 of these 360-degree-folding assemblies and used bobby pins to lock the four CO edges together at the C centers of the diamond-shaped outer faces. Then I used bobby pins again to lock the 30 assemblies together at the 20 convergent A vertexes and the 12 convergent B sphere-surface vertexes. Altogether they made a bigger sphere than the calculated radius, because of the accumulated thickness of the foldings of the construction paper's double-walled (trivalent) interfacing of the 30 internal tetrahedral components. (See Fig. [986.502D](#).)



[Fig. 986.504](#)

986.504 Instead of the just previously described 30 assemblies of four identical spherically central tetrahedra, each with all of their 62 vertexes in the unit-radius spheres, I next decided to make separately the 120 correspondingly convergent (non-arc-edged but chorded) tetrahedra of the tetravolume-5 rhombic triacontahedron, with its 30 flat ABAB diamond faces, the center C of which outer diamond faces is criss-crossed at right angles at C by the short axis A-A of the diamond and by its long axis B-B, all of which diamond bounding and criss-crossing is accomplished by the same 15 greatcircle planes that also described the 30 diamonds' outer boundaries. As noted, the criss-crossed centers of the diamond faces occur at C, and all the C points are at the prime-vector-radius outwardly from the volumetric center O of the rhombic triacontahedron, while OA is 1.07 of vector unity and OB is 1.17 of vector unity outward, respectively, from the rhombic triacontahedron's symmetrical system's center of volume O. (See Figs. [986.504A](#) and [986.504B](#).)

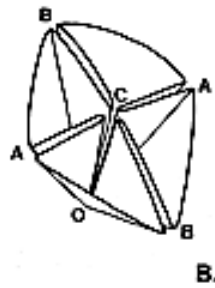


$$\angle COA = 20.90515745^\circ = \text{arc sin} \left(\frac{\sin 18^\circ}{\sin 60^\circ} \right)$$

$$\angle AOB = 37.37736814^\circ = \text{arc sin} \left(\frac{\sin 18^\circ}{\sin 60^\circ \sin 36^\circ} \right)$$

$$\angle BOC = 31.71747441^\circ = \text{arc sin} \left(\frac{\sin 18^\circ}{\sin 36^\circ} \right)$$

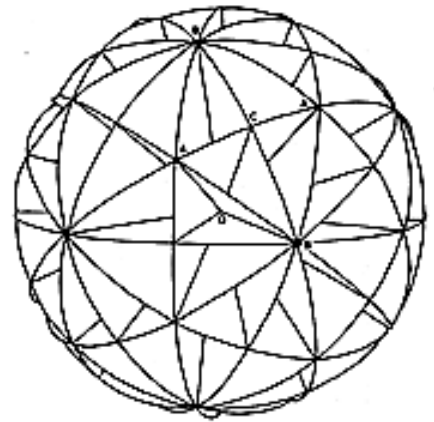
A.



B.



C.



D.

Fig. 986.502 Thirty Great-circle Discs Foldable into Rhombic Triacontahedron System: Each of the four degree quadrants, when folded as indicated at A and B, form separate T Quanta Module tetrahedra. Orientations are indicated by letter on the great-circle assembly at D.

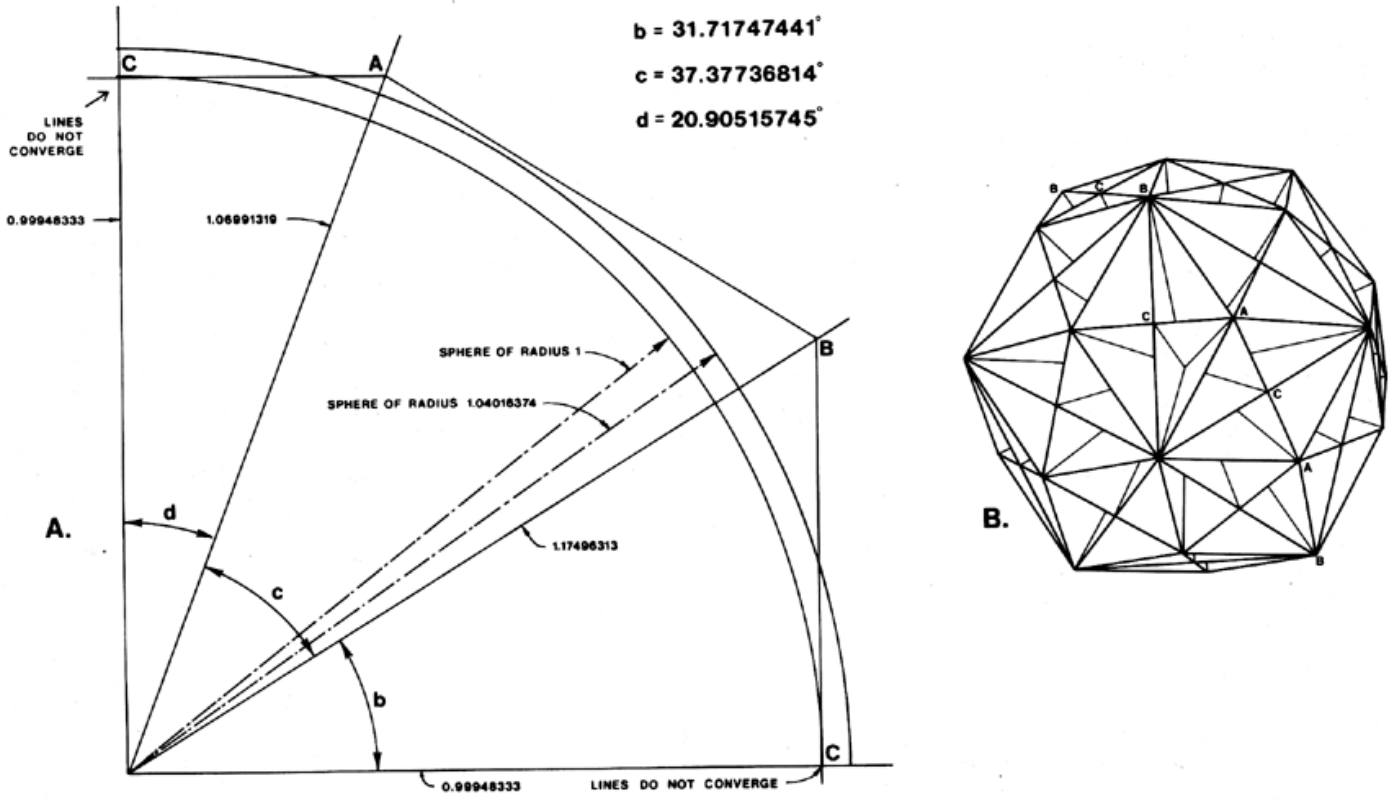
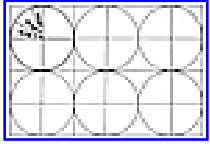


Fig. 986.504 Profile of Quadrants of Sphere and Rhombic Triacontahedron: Central angles and ratios of radii are indicated at A. Orientation of modules in spherical assembly is indicated by letters at B.



[Fig. 986.505](#)

986.505 To make my 120 OABC tetrahedra I happened to be using the same construction paperboard I had used before in making the 30 arc-edged great-circle components. The construction paperboard happened to come in sheets 24 by 36 inches, i.e., two feet by three feet. In making the previously described spherical triacontahedron out of these 24-by-36-inch sheets, I had decided to get the most out of my material by using a 12-inch-diameter circle, so that I could lay out six of them tangentially within the six 12-inch-square modules of the paperboard to produce the 30 foldable great circles. This allowed me to cut out six intertangent great circles from each 24-by-36-inch construction paper sheet. Thirty great circles required only five sheets, each sheet producing six circles. To make the 12 separate T Quanta Module tetrahedra, I again spontaneously divided each of the same-size sheets into six squares with each of the six circles tangent to four edges of each square (Fig. [986.505](#)).

986.506 In starting to make the 120 separate tetrahedra (60 positive, 60 negative—known as T Quanta Modules) with which to assemble the triacontahedron- which is a chord-edged polyhedron vs the previous "spherical" form produced by the folded 15-great-circle patterning—I drew the same 12-inch-edge squares and, tangentially within the latter, drew the same six 12-inch-diameter circles on the five 24-by-36-inch sheets, dividing each circle into four quadrants and each quadrant into three subsections of 20.9 degrees, 37.4 degrees, and 31.7 degrees, as in the T Quanta Modules.

986.507 I planned that each of the quadrants would subsequently be cut from the others to be folded into one each of the 120 T Quanta Module tetrahedra of the triacontahedron. This time, however, I reminded myself not only to produce the rhombic triacontahedron with the same central angles as in the previous spheric experiment's model, but also to provide this time for surfacing their clusters of four tetrahedra ABCO around their surface point C at the mid-crossing point of their 30 flat diamond faces. Flat diamond faces meant that where the sets of four tetra came together at C, there would not only have to be four 90-degree angles on the flat surface, but there would be eight internal right angles at each of the internal flange angles. This meant that around each vertex C corner of each of the four T Quanta Modules OABC coming together at the diamond face center C there would have to be three 90-degree angles.

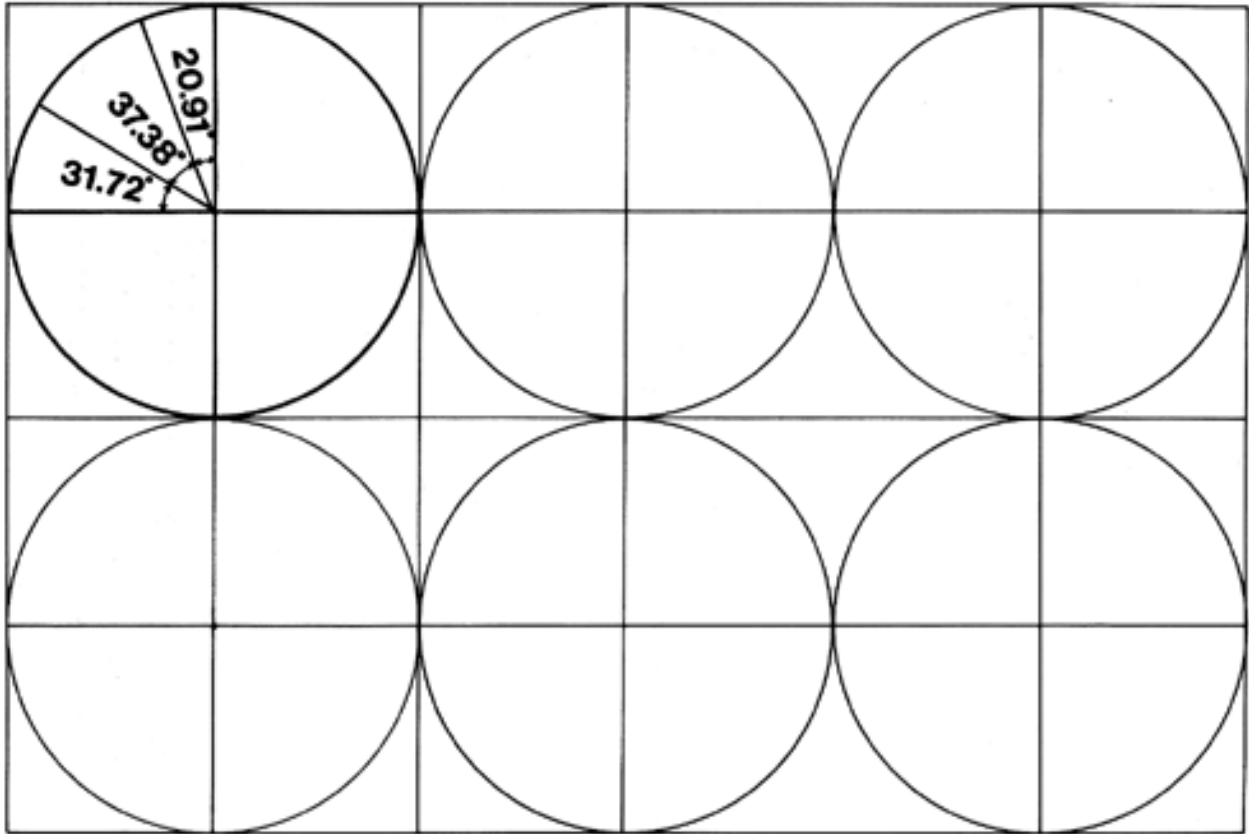
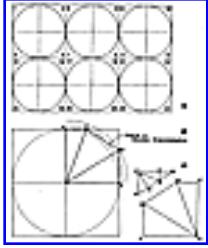


Fig. 986.505 Six Intertangent Great-circle Discs in 12-inch Module Grid: The four 90 degree quadrants are folded at the central angles indicated for the T Quanta Module.

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[Fig. 986.508](#)

986.508 Looking at my "one-circle-per-each-of-six-squares" drawing, I saw that each sheet was divided into 24 quadrant blanks, as in Fig. [986.508A](#)986.508A. Next I marked the centers of each of the six circles as point O, O being the volumetric center of the triacontahedral system. Then I realized that, as trigonometrically calculated, the flat, diamond-centered, right-angled, centrally criss-crossed point C of the triacontahedron's outer faces had to be at our primitive unit-vector-length distance outwardly from the system center O, whereas in the previous arc-edged 30-great-circle-folded model the outer vertex C had been at full- spherical-system-radius distance outwardly from O. In the spherical 15-great-circle-model, therefore, the triacontahedron's mid-flat-diamond-face C would be at 0.07 lesser radial distance outwardly from O than would the diamond corner vertexes A and vertex A itself at a lesser radial distance outwardly from O than diamond corner vertex B. (See Fig. [986.504A](#).)

986.509 Thinking about the C corner of the described tetrahedron consisting entirely of 90-degree angles as noted above, I realized that the line C to A must produce a 90- degree-angle as projected upon the line OC", which latter ran vertically outward from O to C", with O being the volumetric center of the symmetrical system (in this case the rhombic triacontahedron) and with C" positioned on the perimeter exactly where vertex C had occurred on each of the previous arc-described models of the great circles as I had laid them out for my previous 15 great-circle spherical models. I saw that angle ACO must be 90 degrees. I also knew by spherical trigonometry that the angle AOC would have to be 20.9 degrees, so I projected line OA outwardly from O at 20.9 degrees from the vertical square edge OC.

986.510 At the time of calculating the initial layout I made two mistaken assumptions: first, that the 0.9995 figure was critically approximate to 1 and could be read as 1; and second (despite Chris Kitrick's skepticism born of his confidence in the reliability of his calculations), that the 0.0005 difference must be due to the residual incommensurability error of the inherent irrationality of the mathematicians' method of calculating trigonometric functions. (See the Scheherazade Numbers discussed at Sec. [1230](#).) At any rate I could not lay out with drafting tools a difference of 0.0005 of six inches, which is 0.0030 of an inch. No draftsman can prick off a distance even ten times that size. (I continue to belabor these mistaken assumptions and the subsequent acknowledgments of the errors because it is always upon the occasion of my enlightened admission of error that I make my greatest discoveries, and I am thus eager to convey this truth to those seeking the truth by following closely each step of this development,

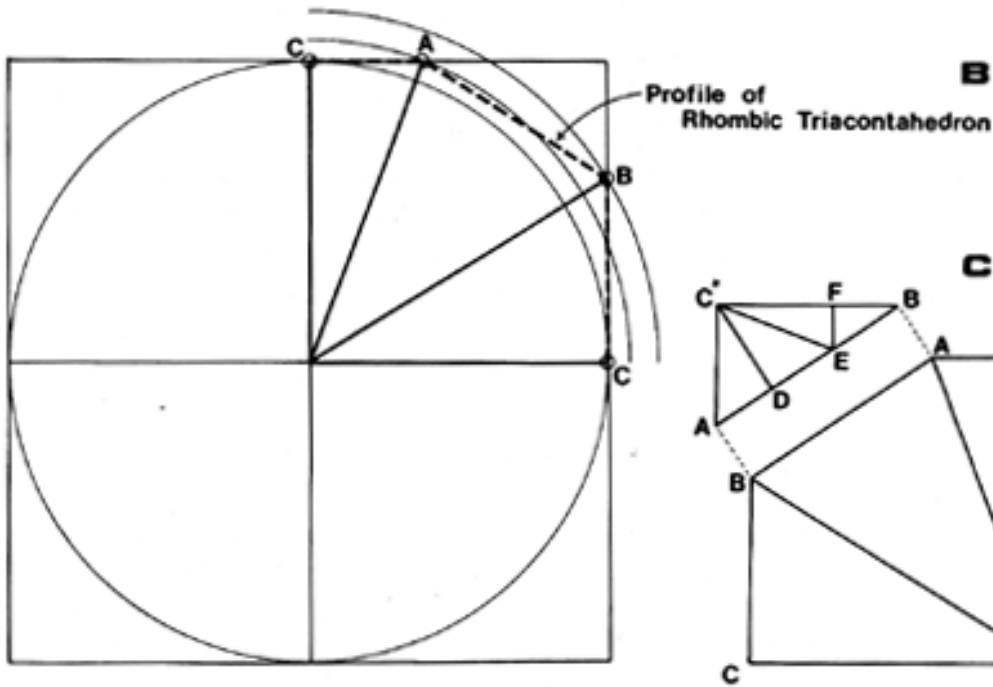
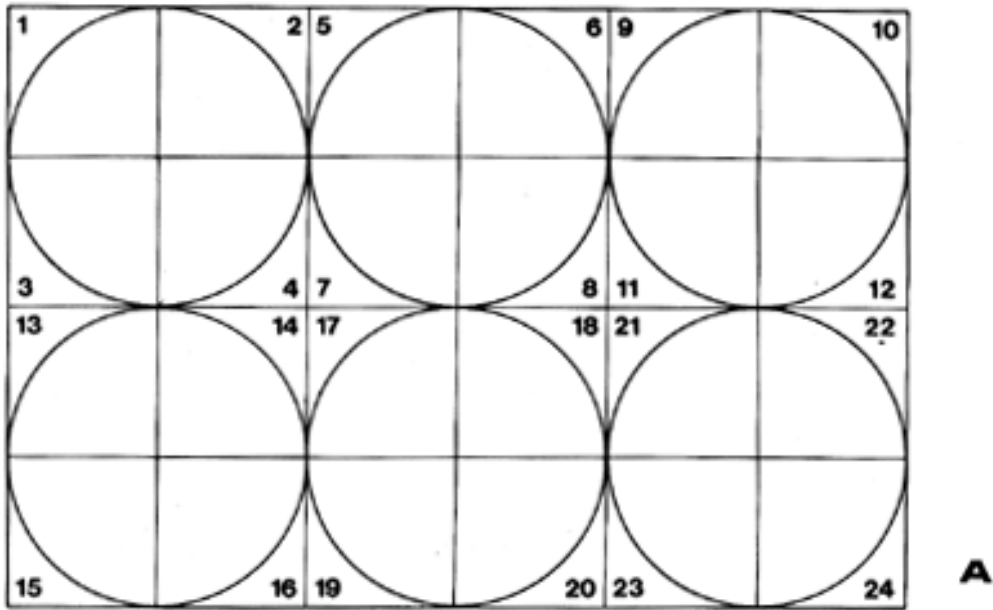


Fig. 986.508 Six Intertangent Great-circle Discs: Twelve-inch module grids divided into 24 quadrant blanks at A Profile of rhombic triacontahedron superimposed on quadrant at B.

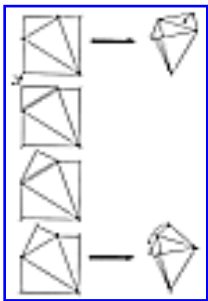
which leads to one of the most exciting of known discoveries.)

986.511 In order to produce the biggest model possible out of the same 24-by-36-inch construction paper blanks, I saw that vertex A of this new T Quanta Module model would have to lie on the same 12-inch circle, projecting horizontally from A perpendicularly (i.e., at right angles), upon OX at C. I found that the point of 90-degree impingement of AC on OX occurred slightly inward (0.041, as we learned later by/trigonometry), vertically inward, from X. The symbol X now occurs on my layout at the point where the previous spherical model's central diamond vertex C had been positioned—on the great-circle perimeter. Trigonometric calculation showed this distance between C and X to be 0.041 of the length of our unit vector radius. Because (1) the distance CO is established by the right-angled projection of A upon OX; and because (2) the length CO is also the prime vector of synergetics' isotropic vector matrix itself, we found by trigonometric calculation that when the distance from O to C is 0.9995 of the prime vector's length, that the tetravolume of the rhombic triacontahedron is exactly 5.

986.512 When the distance from O to C is 0.9995, then the tetravolume of the rhombic triacontahedron is exactly 5. OC in our model layout is now exactly the same as the vector radius of the isotropic vector matrix of our "generalized energy field." OC rises vertically (as the right-hand edge of our cut-out model of our eventually-to-be-folded T Quanta Module's model designing layout) from the eventual triacontahedron's center O to what will be the mid-diamond face point C. Because by spherical trigonometry we know that the central angles of our model must read successively from the right-hand edge of the layout at 20.9 degrees, 37.4 degrees, and 31.7 degrees and that they add up to 90 degrees, therefore line OC' runs horizontally leftward, outward from O to make angle COC' 90 degrees. This is because all the angles around the mid-diamond criss-cross point C are (both externally and internally) 90 degrees. We also know that horizontal OC' is the same prime vector length as vertical OC. We also know that in subsequent folding into the T Quanta Module tetrahedron, it is a mathematical requirement that vertical OC be congruent with horizontal OC' in order to be able to have these edges fold together to be closed in the interior tetrahedral form of the T Quanta Module. We also know that in order to produce the required three 90-degree angles (one surface and two interior) around congruent C and C' of the finished T Quanta Module, the line C'B of our layout must rise at 90 degrees vertically from C' at the leftward end of the horizontal unit vector radius OC'. (See Fig. [986.508C](#).)

986.513 This layout now demonstrates three 90-degree corners with lines OC vertical and OC' horizontal and of the same exact length, which means that the rectangle COC'C" must be a square with unit-vector-radius edge length OC. The vertical line C'C" rises from C' of horizontal OC' until it encounters line OB, which—to conform with the triacontahedron's interior angles as already trigonometrically established—must by angular construction layout run outwardly from O at an angle of 31.7 degrees above the horizontal from OC' until it engages vertical C'C" at B. Because by deliberate construction requirement the angle between vertical OC and OA has been laid out as 20.9 degrees, the angle AOB must be 37.4 degrees—being the remainder after deducting both 20.9 degrees and 31.7 degrees from the 90-degree angle lying between vertical OC and horizontal OC'. All of this construction layout with OC' horizontally equaling OC vertically, and with the thus-far-constructed layout's corner angles each being 90 degrees, makes it evident that the extensions of lines CA and C'B will intersect at 90 degrees at point C", thus completing the square OC`C"C of edge length OC, which length is exactly 0.999483332 of the prime vector of the isotropic vector matrix's primitive cosmic- hierarchy system.

986.514 Since ACO, COC', and OC'B are all 90-degree angles, and since vertical CO = horizontal C'O in length, the area COC'C" must be a square. This means that two edges of each of three of the four triangular faces of the T Quanta Module tetrahedron, and six of its nine prefolded edges (it has only six edges after folding), are congruent with an exactly square paperboard blank. The three triangles OCA, OAB, and OBC' will be folded inwardly along AO and BO to bring the two CO and CO' edges together to produce the three systemically interior faces of the T Quanta Module.



[Fig. 986.515](#)

986.515 This construction method leaves a fourth right-triangular corner piece AC"B, which the dividers indicated—and subsequent trigonometry confirmed—to be the triangle exactly fitting the outer ABC-triangular-shaped open end of the folded-together T Quanta Module OABC. O" marks the fourth corner of the square blank, and trigonometry showed that C"A = C'B and C"B = AC, while AB of triangle OBA by construction is congruent with AB of triangle AC"B of the original layout. So it is proven that the vector- edged square COC'C" exactly equals the surface of the T Quanta Module tetrahedron CABO. (See [Fig. 986.515](#).)

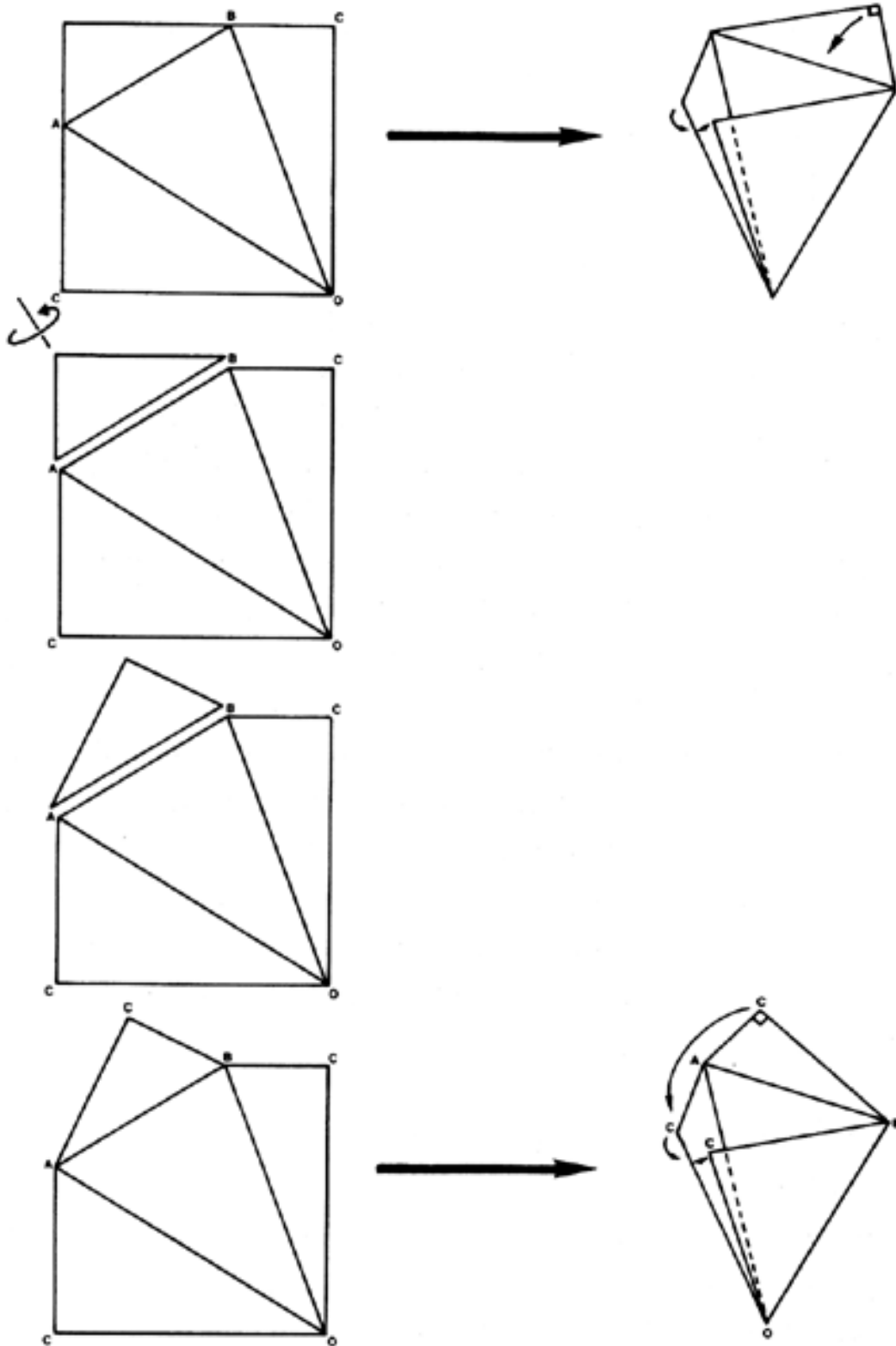


Fig. 986.515 T Quanta Module Foldable from Square: One of the triangular corners may be hinged and reoriented to close the open end of the folded tetrahedron.

986.516 The triangle $AC''B$ is hinged to the T Quanta Module along the mutual edge AB , which is the hypotenuse of the small $AC''B$ right triangle. But as constructed the small right triangle $AC''B$ cannot be hinged (folded) to close the T Quanta Module tetrahedron's open-end triangular area ABC —despite the fact that the hinged-on triangle $AC''B$ and the open triangle ABC are dimensionally identical. $AC''B$ is exactly the right shape and size and area and can be used to exactly close the outer face of the T Quanta Module tetrahedron, *if*—but only if—it is cut off along line BA and is then turned over so that its faces are reversed and its B corner is now where its A corner had been. This is to say that if the square $COC''C''$ is made of a cardboard sheet with a red top side and a gray underside, when we complete the tetrahedron folding as previously described, cut off the small corner triangle $AC''B$ along line BA , reverse its face and its acute ends, and then address it to the small triangular ABC open end of the tetrahedron $CABO$, it will fit exactly into place, but with the completed tetrahedron having three gray faces around vertex O and one red outer face CAB . (See Fig. [986.508C](#).)

986.517 Following this closure procedure, when the $AC''B$ triangles of each of the squares are cut off from $COC''C''$ along line AB , and right triangle $AC''B$ is reversed in face and its right-angle corner C'' is made congruent with the right-angle corner C of the T Quanta Module's open-end triangle, then the B corner of the small triangle goes into congruence with the A corner of the open-end triangle, and the A corner of the small triangle goes into congruence with the B corner of the open-end triangle—with the 90-degree corner C becoming congruent with the small triangle's right-angle corner C'' . When all 120 of these T Quanta Module tetrahedra are closed and assembled to produce the triacontahedron, we will have all of the 360 gray faces inside and all of the 120 red faces outside, altogether producing an externally red and an internally gray rhombic triacontahedron.

986.518 In developing the paper-folding pattern with which to construct any one of these 120 identical T Quanta Module tetrahedra, we inadvertently discovered it to be foldable out of an exact square of construction paper, the edge of which square is almost (0.9995 of the prime vector 1) identical in length to that of the prime vector radius of synergetics' closest-packed unit-radius spheres, and of the isotropic vector matrix, and therefore of the radii and chords of the vector equilibrium—which synergetics' vector (as with all vectors) is the product of mass and velocity. While the unit-vector length of our everywhere-the-same energy condition conceptually idealizes cosmic equilibrium, as prime vector (Sec. [540.10](#)) it also inherently represents everywhere-the-same maximum cosmic velocity unfettered in vacuo—ergo, its linear velocity (symbolized in physics as lower-case c) is that of all radiation—whether beamed or piped or linearly focused—the velocity of whose unbeamed, omnidirectionally outward, surface growth rate always amounts to the second-powering of the linear speed. Ergo, omniradiance's wave surface growth rate is c^2 .

986.519 Since the edge length of the exactly 5.0000 (alpha) volumed T Quanta Module surface square is 0.9995 of the prime vector 1.0000 (alpha), the surface-field energy of the T Quanta Module of minimum energy containment is $0.9995 V^2$, where 1.0000 (alpha) V is the prime vector of our isotropic vector matrix. The difference—0.0005—is minimal but not insignificant; for instance, the mass of the electron happens also to be 0.0005 of the mass of the proton.

[Next Section: 986.520](#)
