

1053.00 **Superficial and Volumetric Hierarchies**

1053.10 **Spherical Triangular Grid Tiles:** The interrelationship of the vector equilibrium and the icosahedron when their respective 25- and 31-great-circle grids are superimposed on one another, with the center of area of the vector equilibrium's eight spherical triangles congruent with the areal centers of eight of the icosahedron's 20 spherical triangles, reveals a fundamental, asymmetrical, six-axis, alternative, impulsive- pulsative potential of surface intertransformabilities in respect to which the vector equilibrium serves as the zero between the positive and negative, "relative" asymmetry, deviations.

1053.11 The vector equilibrium's 25 and the icosahedron's 31 spherical-great-circle grids manifest different least-common-denominator, identically angled, spherical triangular "tiles," which together exactly cover and subdivide the spherical surface in whole even numbers of tiles; the vector equilibrium having 48 such LCD triangles and the icosahedron having $2\frac{1}{2}$ times as many LCD triangles, i.e., 120.

1053.12 The fundamental fiveness of the icosahedron is split two ways, with $2\frac{1}{2}$ going one way (the outside-out way) and $2\frac{1}{2}$ going the other way (the inside-out way). The least-common-denominator triangular surface subdivision of the vector equilibrium's sphere provides 48 angularly identical (24 inside-out and 24 outside-out) subdivisions as spherical surface "tiles" that exactly cover one sphere.

1053.13 $120/48 = 2\frac{1}{2}$; and there are always both the four *positively* skew-rotated and the four *negatively* skew-rotated sets of spherical triangles (two sets of four each), symmetrically borrowed from among the spherical total of 20 equiangular, spherical triangles of each of two spherical icosahedra (each of radius 1)_which four out of 20 ($20/4 = 5$) spherical icosahedron's triangles' centers of area are exactly concentrically registerable upon every other one of the spherical octahedron's eight triangles, which areal centers of the octahedron's eight triangles are also always concentrically and symmetrically in register with the eight equiangular, spherical triangles of the spherical vector equilibrium when the octahedron and the vector equilibrium spheres are all of the same unity-1 radius. With this registration of four out of eight centers of the icosahedron upon the octahedron-vector equilibrium's eight triangular surfaces each, we find that one icosahedron set of four skews rotationally positive, while the set of four from another icosahedron phase registers the negative skew rotation, which is a +30 degrees or -30 degrees circumferentially-away-from-zero, rotational askewness for a total of 60 degrees differential between the extremes of both. The remaining 16 out of the total of 20 triangles of each of the two different (plus-or-minus-30-degree) phase

icosahedra, subdivide themselves in four sets of four each, each of which sets of four arrange themselves in polarized symmetry upon each of the octahedron's four other spherical triangles which are not concentrically occupied by either the positively- or negatively-skew, concentric sets, of four each, triangles, neither of which four sets of four each non-triangularly-concentric sets repeat the other sets' complementary, asymmetric but polarized, array in superimposition upon the octahedron's four nonconcentrically occupied triangles.

1053.14 It was in discovering this alternate, concentric askewness of icosagon-octa, however, that we also learned that the symmetrical, equiangular, spherical triangle areas, filled evenly—but rotationally askew—with sets of 15 of the icosahedron's 120 LCD triangles, exactly registered with the spherical surface area of one of the spherical octahedron's eight triangular faces (each of which are bound by 90-degree corners and 90-degree arc edges). This meant, however, that the 15 LCD icosagon triangles' plus-rotated askew phases are not congruent with one another but are superimposed in alternately askewed arrays, both in the cases of the four concentric triangles and in the cases of the nonconcentrically-registered triangles.

1053.15 Because each of the octahedron's eight faces is subdivided by its respective six sets of spherical "right" triangles (three positive—three negative), whose total of $6 \times 8 = 48$ triangles are the 48 LCD's vector-equilibrium, symmetric-phase triangles, and because $120/48 = 2 \frac{1}{2}$, it means that each of the vector equilibrium's 48 triangles has superimposed upon it $2 \frac{1}{2}$ positively askew and $2 \frac{1}{2}$ negatively askew triangles from out of the total inventory of 120 LCD asymmetric triangles of each of the two sets, respectively, of the two alternate phases of the icosahedron's limit of rotational aberrating of the vector equilibrium. This $2 \frac{1}{2}$ positive superimposed upon the $2 \frac{1}{2}$ negative, 120-LCD picture is somewhat like a Picasso duo-face painting with half a front view superimposed upon half a side view. It is then in transforming from a positive two-and-one-halfness to a negative two-and-one-halfness that the intertransformable vector-equilibrium-to-icosahedron, icosahedron-to-vector-equilibrium, equilibrium-to-disequilibrium attains sumtotally and only dynamically a spherical *fiveness* (see Illus. [982.61](#) in color section).

1053.16 This half-in-the-physical, half-in-the-metaphysical; i.e., half-conceptual, half- nonconceptual; i.e., now you see it, now you don't—and repeat, behavior is characteristic of synergetics with its nuclear sphere being both concave and convex simultaneously, which elucidates the microcosmic, turn-around limit of Universe as does the c^2 the spherical-wave-terminal-limit velocity of outwardness elucidate the turn-around-and- return limit of the macrocosm.

1053.17 This containment of somethingness by uncontained nothingness: this split personality $+2\ 1/2, -2\ 1/2; +5, -5, +0, -0$; plural unity: this multiplicative twoness and additive twoness of unity; this circumferential-radial; this birth-death, birth-death; physical- metaphysical, physical-metaphysical; yes-no, yes-no-ness; oscillating-pulsating geometrical intertransformability field; Boltzmann importing-exporting elucidates the a priori nature of the associative-disassociative; entropic-syntropic; energetic-synergetic inherency of cosmic discontinuity with its ever locally renewable cyclic continuities, wherewith Universe guarantees the eternally regenerative scenario integrity.

1053.20 **Platonic Polyhedra:** There are 48 spherical triangular tiles of the vector equilibrium nuclear sphere, which 48 triangles' pattern can be symmetrically subdivided into five different sets of symmetrical interpatterning which coincide exactly with the projection outward onto a sphere of the five omnisymmetrical planar-defined Platonic polyhedra, whose linear edges are outlined by the respective chords of the congruent vector equilibrium's symmetrical 25-great-circle grid and the icosahedron's 31-great-circle grid. These equiedged Platonic solids are the icosahedron, the octahedron, the cube, the tetrahedron, and the regular dodecahedron. (The vector equilibrium is one of the Archimedean polyhedra; it was called *cuboctahedron* by the Greeks.)

1053.21 The chords of these five spherical geometric integrities all interact to produce those well-known equiedged polyhedra commonly associated with Plato. The intervolumetric quantation of these five polyhedra is demonstrated as rational when referenced to the tetrahedron as unity. Their surface values can also be rationally quantized in reverse order of magnitude by the 48 spherical triangle tiles in whole, low- order, even numbers. These hierarchies are a discovery of synergetic geometry.

1053.30 **LCD Superficial Quantation of Systems:** Because the icosahedron's 31- great-circle grid discloses 120 least-common-denominator, spherical triangular, whole tiling units, we require a special-case, least-common-surface-denominator identity as a name for the 48 spherical tiles of the vector equilibrium. The 120 spherical surface triangular tiles (60 insideout and 60 outside-out) do indeed constitute the least-common- spherical and planar polyhedra's whole-surface denominators, ergo LCDs, of all closed systems; for all systems are either simplex (atomic) or complex (molecular) manifests of polyhedra. All systems, symmetrical or asymmetrical, have fundamental insidiness (micro) and outsidiness (macro) irrelevancies that leave the residual-system relevancies accountable as topological characteristics of the polyhedra.

1053.31 As we have learned elsewhere, the sphere, as demonstrated by the spherical icosahedral subdivisions, discloses a different least-common-denominator spherical subdivision in which there are 120 such tiles (60 positive and 60 negative), which are generalizable mathematically as the least-common surface denominator of surface unity, ergo, of systems in general superficially quantated. Because the icosahedron provides the maximum asymmetries into which the vector equilibrium's universally zero-balanced surface can be transformed, and since the effect of the icosahedron—which introduces the prime number five into Universe systems—is one of transforming, or splitting, equilibrium two ways, we find time after time that the interrelationship of the vector equilibrium and the icosahedron surfaces to be one such elegant manifestation of the number $2\frac{1}{2}$ — $2\frac{1}{2}$ positive and $2\frac{1}{2}$ negative, of which the icosahedron's fiveness consists. This half- positive and half-negative dichotomization of systems is the counterpart in pure principle of the nuclear accounting that finds that the innermost ball of the closest-packed symmetrical aggregate always belongs half to a positive world and half to a negative world; that is, the inbound half (implosive) and the outbound half (explosive) altogether make a kinetically regenerative whole centrality that never belongs completely to either world.

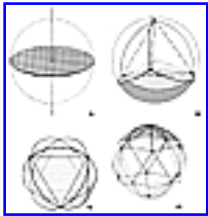
1053.32 It is a condition analogous to the sphere with its always and only complementarity of insidiness and outsidiness, convexity and concavity. A sphere may be thought of as half concave and half convex as well as having two different poles.

1053.33 For the moment, considering particularly spherical-system surfaces, we find the same $2\frac{1}{2}$ -ness relationship existing between the vector equilibrium and the icosahedron, with their respective least common denominator's surface triangle building tiles (of which the vector equilibrium's 48 LCDs have five of the equiedged Platonic solids and the icosahedron's 120 LCDs have two of the equiedged Platonic solids). The icosahedron-coexistent pentagonal dodecahedron is the special-case system of domains of the icosahedron's 12 vertexes; it is not a structure in its own right. Plato's five omniequifaceted, equiedged and -vertexed, "solids" were the cube, tetrahedron, octahedron, icosahedron, and dodecahedron. All five of these solids are rationally accounted by the LCD spherical surface triangular tilings of the vector equilibrium and the icosahedron.

1053.34 The icosahedron has 120 triangles (60+, 60-), which are the least common denominators of spherical surface unity of Universe; ergo, so important as to have generated, for instance, the ancient Babylonians' adoption of 60 both for increments of time and for circular mensuration. The Babylonians attempted to establish a comprehensive coordinate mensural system that integrated time and matter. Their artifacts show that they had discovered the 60 positive and 60 negative, 120 spherical right triangles of spheres. That their sixtyness did not uncover nature's own rational coordinate system should not be permitted to obscure the fact that the Babylonians were initiating their thinking systematically in polyhedral spherical wholeness and in 60-degree vs. 90-degree coordination, which was not characteristic of the geometrical exploration of a later date by the Egyptians and Greeks, who started very locally with lines, perpendiculars, and planes.

1053.35 The great $2\frac{1}{2}$ transformation relations between the vector equilibrium and the icosahedron once again manifest in surface equanimity as the LCD surface triangular tiling, which is $2\frac{1}{2}$ times 48, or 120.

1053.36 **Sphere: Volume-surface Ratios:** The largest number of similar triangles into which the whole surface of a sphere may be divided is 120. (See Secs. [905](#) and [986](#).) The surface triangles of each of these 120 triangles consist of one angle of 90 degrees, one of 60 degrees, and one of 36 degrees. Each of these 120 surface triangles is the fourth face of a similar tetrahedron whose three other faces are internal to the sphere. Each of these tetra has the same volume as have the A or B Quanta Modules. Where the tetra is 1, the volume of the rhombic triacontahedron is approximately 5. Dividing 120 by 5 = 24 = quanta modules per tetra. The division of the rhombic triacontahedron of approximately tetravolume-5 by its 120 quanta modules discloses another unit system behavior of the number 24 as well as its appearance in the 24 external vector edges of the VE. (See Sec. [1224.21](#))



[Fig. 1053.37](#)

1053.37 Since the surface of a sphere exactly equals the internal area of the four great circles of the sphere, and since the surface areas of each of the four triangles of the spherical tetrahedron also equal exactly one-quarter of the sphere's surface, we find that the surface area of one surface triangle of the spherical tetrahedron exactly equals the internal area of one great circle of the sphere; wherefore

1 spherical tetra's triangle	= 1 great circle
2 spherical octa's triangles	= 1 great circle
5 spherical icosa's triangles	= 1 great circle
30 spherical Basic LCD triangles	= 1 great circle

1053.40 **Superficial Hierarchy:** We have here a total spherical surface subdivisioning hierarchy predicated upon (a) the relative number of LCD (48/n) tiles necessary to define each of the following's surface triangles, wherein the tetrahedron requires 12; the octahedron 6; cube 8; and rhombic dodecahedron 4; in contradistinction to (b) their respective volumetric quantations expressed in the terms of the planar-faceted tetrahedron as unity.

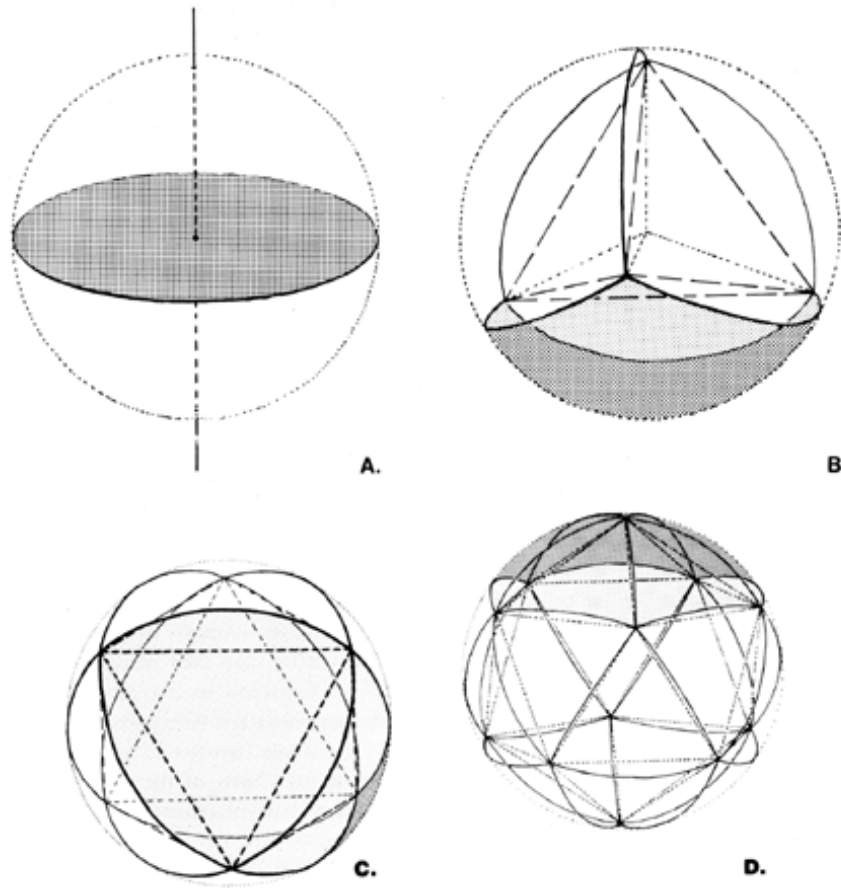


Fig. 1053.37 Spherical Great Circles Are Commensurable with Spherical Triangles of Three Prime Structural Systems: Tetra Octa and Icosa:

- A. Internal area of one great circle of a sphere
- B. Surface triangle of spherical tetrahedron
- C. Surface triangle of spherical octahedron
- D. Surface triangle of spherical icosahedron

[1 great circle = 1 tetra triangle = 2 octa triangles = 5 icosa triangles = 30 basic LCD triangles]

1053.41 **Table: Spherical Surface Hierarchy**

<i>Number of Spherical LCD (48 VE)</i>	<i>Spherical Conformation</i>		<i>Nuclear Sphere's Radius-1 Volumetric Hierarchy</i>
48	define one		
	Vector Equilibrium sphere	×2=96	1
12	define one		
	Tetrahedron face	×2=24	3
8	define one		
	Cube face	×2=16	4
6	define one		
	Octahedron face	×2=12	6
4	define one		
	Rhombic Dodecahedron face	×2=8	
4	define one		
	Regular Dodecahedron face	×2=8	
2 1/2	define one		
	Icosahedron face	×2=5	

1053.50 **Volumetric Hierarchy:** With a nuclear sphere of radius-1, the volumetric hierarchy relationship is in reverse magnitude of the superficial hierarchy. In the surface hierarchy, the order of size reverses the volumetric hierarchy, with the tetrahedron being the largest and the rhombic dodecahedron the smallest.

1053.51 **Table: Volumetric Hierarchy:** The space quantum equals the space domain of each closest-packed nuclear sphere:

Space quantum	= 1
Tetrahedron	= 1
Nuclear vector equilibrium	= 2 1/2
Nuclear icosahedron	= 2 1/2
Cube	= 3
Octahedron	= 4 ¹⁰
Nuclear sphere	= 5
Rhombic dodecahedron	= 6

(Footnote 10: The octahedron is always double, ergo, its fourness of volume is its prime number manifest of two, which synergetics finds to be unique to the octahedron.)

1053.51A **Table: Volumetric Hierarchy (revised):** The space quantum equals the space domain of each closest-packed nuclear sphere:

Space quantum	= 1
Tetrahedron	= 1
Nuclear vector equilibrium	= 2 1/2
Nuclear icosahedron	= 2 1/2
Cube	= 3
Octahedron	= 4
Rhombic triacontahedron	= 5+
Rhombic dodecahedron	= 6

1053.60 **Reverse Magnitude of Surface vs. Volume:** Returning to our consideration of the reverse magnitude hierarchy of the surface vs. volume, we find that both embrace the same hierarchical sequence and have the same membership list, with the icosahedron and vector equilibrium on one end of the scale and the tetrahedron on the other. The tetrahedron is the smallest omnisymmetrical structural system in Universe. It is structured with three triangles around each vertex; the octahedron has four, and the icosahedron has five triangles around each vertex. We find the octahedron in between, doubling its prime number twoness into volumetric fourness, as is manifest in the great-circle foldability of the octahedron, which always requires two sets of great circles, whereas all the other icosahedron and vector equilibrium 31 and 25 great circles are foldable from single sets of great circles .

1053.601 **Octahedron:** The octahedron—both numerically and geometrically—should always be considered as quadrivalent; i.e., congruent with self; i.e., doubly present. In the volumetric hierarchy of prime-number identities we identify the octahedron's prime-number twoness and the inherent volume-fourness (in tetra terms) as volume 22, which produces the experiential volume 4.

1053.61 The reverse magnitudes of the surface vs. volume hierarchy are completely logical in the case of the total surface subdivision starting with system totality. On the other hand, we begin the volumetric quantation hierarchy with the tetrahedron as the volumetric quantum (unit), and in so doing we build from the most common to the least common omnisymmetrical systems of Universe. In this system of biggest systems built of smaller systems, the tetrahedron is the smallest, ergo, most universal. Speaking holistically, the tetrahedron is predominant; all of this is analogous to the smallest chemical element, hydrogen, being the most universally present and plentiful, constituting the preponderance of the relative abundance of chemical elements in Universe.

1053.62 The tetrahedron can be considered as a whole system or as a constituent of systems in particular. It is the particulate.

1053.70 **Container Structuring: Volume-surface Ratios**

1053.71 When attempting to establish an international metric standard of measure for an integrated volume-weight unit to be known as "one gram" and deemed to consist of one cubic centimeter of water, the scientists overlooked the necessity for establishing a constant condition of temperature for the water. Because of expansion and contraction under changing conditions of temperature a constant condition of 4 degrees centigrade was later established internationally. In much the same way scientists have overlooked and as yet have made no allowance for the inherent variables in entropic and syntropic rates of energy loss or gain unique to various structurally symmetrical shapes and sizes and environmental relationships. (See Sec. [223.80](#), "Energy Has Shape.") Not only do we have the hierarchy of relative volume containments respectively of equiedged tetra, cube, octa, icosahedron, "sphere," but we have also the relative surface-to-volume ratios of those geometries and the progressive variance in their relative structural-strength-to-surface ratios as performed by flat planes vs simple curvature; and as again augmented in strength out of the same amount of the same material when structured in compound curvature.

1053.72 In addition to all the foregoing structural-capability differentials we have the tensegrity variables (see [Chap. 7](#)), as all these relate to various structural capabilities of various energy patternings as containers to sustain their containment of the variously patterning contained energies occurring, for instance, as vacuum vs crystalline vs liquid vs gaseous vs plasmic vs electromagnetic phases; as well as the many cases of contained explosive and implosive forces. Other structural variables occur in respect to different container-contained relationships, such as those of concentrated vs distributive loadings under varying conditions of heat, vibration, or pressure; as well as in respect to the variable tensile and compressive and shear strengths of various chemical substances used in the container structuring, and their respective heat treatments; and their sustainable strength-time limits in respect to the progressive relaxing or annealing behaviors of various alloys and their microconstituents of geometrically variant chemical, crystalline, structural, and interproximity characteristics. There are also external effects of the relative size- strength ratio variables that bring about internal interattractiveness values in the various alloys as governed by the second-power rate, i.e., frequency of recurrence and intimacy of those alloyed substances' atoms.

1053.73 As geometrical systems are symmetrically doubled in linear dimension, their surfaces increase at a rate of the second power while their volumes increase at a third- power rate. Conversely, as we symmetrically halve the linear dimensions of geometrical systems, their surfaces are reduced at a second-root rate, while their volumes decrease at a third-root rate.

1053.74 A cigar-shaped piece of steel six feet (72 inches) long, having a small hole through one end and with a midgirth diameter of six inches, has an engineering slenderness ratio (length divided by diameter) of 12 to 1: It will sink when placed on the surface of a body of water that is more than six inches deep. The same-shaped, end-pierced piece of the same steel of the same 12-to-1 slenderness ratio, when reduced symmetrically in length to three inches, becomes a sewing needle, and it will float when placed on the surface of the same body of water. Diminution of the size brought about so relatively mild a reduction in the amount of surface of the steel cigar-needle's shape in respect to the great change in volume—ergo, of weight—that its shape became so predominantly "surface" and its relative weight so negligible that only the needle's surface and the atomic-intimacy- produced surface tension of the water were importantly responsible for its interenvironmental relationship behaviors.

1053.75 For the same reasons, grasshoppers' legs in relation to a human being's legs have so favorable a volume-to-surface-tension relationship that the grasshopper can jump to a height of 100 times its own standing height (length) without hurting its delicate legs when landing, while a human can jump and fall from a height of only approximately three times his height (length) without breaking his legs.

1053.76 This same volume-to-surface differential in rate of change with size increase means that every time we double the size of a container, the contained volume increases by eight while the surface increases only fourfold. Therefore, as compared to its previous half-size state, each interior molecule of the atmosphere of the building whose size has been symmetrically doubled has only half as much building surface through which that interior molecule of atmosphere can gain or lose heat from or to the environmental conditions occurring outside the building as conductively transferable inwardly or outwardly through the building's skin. For this reason icebergs melt very slowly but accelerate progressively in the rate of melting. For the same reason a very different set of variables governs the rates of gain or loss of a system's energy as the system's size relationships are altered in respect to the environments within which they occur.

1053.77 As oil tankers are doubled in size, their payloads grow eightfold in quantity and monetary value, while their containing hulls grow only fourfold in quantity and cost. Because the surface of the tankers increases only fourfold when their lengths are doubled and their cargo volume increases eightfold, and because the power required to drive them through the sea is proportional to the ship's surface, each time the size of the tankers is doubled, the cost of delivery per cargo ton, barrel, or gallon is halved. The last decade has seen a tenfolding in the size of the transoceanic tankers in which both the cost of the ship and the transoceanic delivery costs have become so negligible that some of the first such shipowners could almost afford to give their ships away at the end of one voyage. As a consequence they have so much wealth with which to corrupt international standards of safety that they now build them approximately without safety factors—ergo, more and more oil tanker wrecks and spills.

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